SOME PROBLEMS ON RICCI SYMMETRIC P-SASAKIAN MANIFOLD

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ABSTRACT:

The purpose of the present paper is to delineate the theory of Ricci symmetric P-Sasakian manifold. Section 2 deals with the Ricci-recurrent P-Sasakian Manifold. In section 3 we have defined and studied Ricci symmetric P-Sasakian manifold. The investigation of projective symmetric P-Sasakian manifold has been delineated in the section 4. In this very section, several possible new results are obtained.

Keywords:

Ricci-recurrent, Ricci-symmetric, P-Sasakian manifold, projective symmetric, Riemannian manifold.

1. INTRODUCTION

We consider an n-dimensional differentiable manifold M^n with a positive definite metric $g_{\alpha\beta}$ which admits a unit covariant vector field η_α satisfying

(1.1) $\nabla_\alpha\eta_\beta - \nabla_\beta\eta_\alpha = 0$

and

(1.2) $\nabla_\alpha\nabla_\beta\eta_\gamma = -\left(g_{\alpha\beta}\eta_\gamma - g_{\alpha\gamma}\eta_\beta\right) + 2\eta_\alpha\eta_\beta\eta_\gamma,$

wherein ∇_α denotes the operator of covariant differentiation with regard to $g_{\alpha\beta}$, such a space M^n is called P-Sasakian manifold[2].

It is easy to verify that in a P-Sasakian manifold, the following relations holds good[1,4]:

(1.3) $\eta_\alpha = g_{\alpha\beta}\,\xi^\beta,$

(1.4) $\eta_\alpha\xi^\alpha = 1,$

(1.5) $g^{\alpha\beta}\eta_\beta = \xi^\alpha,$

(1.6) $g^{\alpha\beta}\eta_\alpha\eta_\beta = 1,$

(1.7) $g_{\alpha\beta}\xi^\alpha\xi^\beta = 1,$

(1.8) $\phi_{\alpha\beta} = \nabla_\alpha\eta_\beta,$

(1.9) $\phi^\alpha_\beta = \nabla_\beta\xi^\alpha,$

(1.10) $\phi^\alpha_\beta\xi^\beta = 0,$

(1.11) $\phi^\alpha_\beta\eta_\alpha = 0,$

(1.12) $\phi^\alpha_\beta\phi^\beta_\gamma = -\,\delta^\alpha_\gamma + \eta_\gamma\xi^\alpha,$

(1.13) $g_{\alpha\beta}\phi^\beta_\gamma = g_{\alpha\gamma} - \eta_\alpha\eta_\gamma,$

(1.14) $\eta_\lambda\, R^\lambda_{\alpha\beta\gamma} = g_{\alpha\gamma}\eta_\beta - g_{\beta\gamma}\eta_\alpha,$

(1.15) $\eta_\lambda\, R^\lambda_\alpha = -(n-1)\eta_\alpha,$

(1.16) $\xi^\alpha\, R^\lambda_\alpha = -(n-1)\xi^\lambda$

and

(1.17) $R^{\epsilon}_{\gamma\beta\alpha} + R^{\epsilon}_{\beta\alpha\gamma} + R^{\epsilon}_{\alpha\gamma\beta} = 0.$

2. RICCI-RECURRENT P-SASAKIAN MANIFOLD

Definition 2.1:

A P-Sasakian manifold whose Ricci tensor $R_{\beta\alpha}$ is recurrent is termed as Ricci-recurrent P-Sasakian manifold.

In view of the definition it follows that [5]:

(2.1) $\nabla_\gamma R_{\beta\alpha} = a_\gamma R_{\beta\alpha}.$

Where a_γ is a non-zero vector field.

Contracting of equation (2.1) with $R^{\beta\alpha} = g^{\beta\tau} g^{\alpha\rho} R_{\tau\rho}$ yields

(2.2) $\nabla_\gamma\left(R_{\tau\rho}R^{\tau\rho}\right) = 2a_\gamma R_{\tau\rho}R^{\tau\rho},$

In consequence of this, we obtain

(2.3) $\left(R_{\tau\rho}R^{\tau\rho}\right)\left(\nabla_\delta a_\gamma - \nabla_\gamma a_\delta\right) = 0,$

As a result of (2.3), we have either

(2.4) $R_{\tau\rho}R^{\tau\rho} = 0$

or

(2.5) $\nabla_\delta a_\gamma = \nabla_\gamma a_\delta$

that is, the recurrence parameter or recurrence vector is gradient.

In view of the relations (2.1) and (2.5), we obtain

(2.6) $\nabla_\delta \nabla_\gamma R_{\beta\alpha} - \nabla_\gamma \nabla_\delta R_{\beta\alpha} = 0,$

Ricci identity yields

(2.7) $R_{\beta\rho}R^{\rho}_{\delta\gamma\alpha} - R_{\alpha\rho}R^{\rho}_{\delta\gamma\beta} = 0,$

Contracting equation (2.7) by $\xi^\delta \xi^\gamma$ yields

(2.8) $R_{\beta\alpha} - R_{\beta\rho}\eta_\alpha\xi^\rho + R_{\alpha\beta} - R_{\alpha\rho}\eta_\beta\xi^\rho = 0.$

In view of above discussion, we have following theorem:

Theorem 2.1:

If ξ^ρ is a non-zero vector field and Ricci tensor is skew-symmetric then the following relation $R_{\beta\rho}\eta_\alpha + R_{\alpha\rho}\eta_\beta = 0$ holds good.

Proof:

If $R_{\alpha\beta}$ is skew-symmetric i.e. $R_{\alpha\beta} = -R_{\beta\alpha}$.

On making use of this into the equation (2.8) follows

(2.9) $[R_{\beta\rho}\eta_{\alpha} + R_{\alpha\rho}\eta_{\beta}]\xi^{\rho} = 0,$

Consequently, follows

(2.10) $R_{\beta\rho}\eta_{\alpha} + R_{\alpha\rho}\eta_{\beta} = 0.$

3. RICCI SYMMETRIC P-SASAKIAN MANIFOLD

Definition 3.1:

If the Ricci tensor $R_{\beta\alpha}$ satisfies the relation

(3.1) $\nabla_{\gamma}R_{\beta\alpha} = 0$

then the manifold is called Ricci symmetric P-Sasakian manifold [5].

By virtue of equations (1.5), (1.9) and (1.14), we obtain

(3.2) $\nabla_{\beta}R_{\alpha}^{\epsilon}\eta_{\epsilon} + \phi_{\beta\epsilon}R_{\alpha}^{\epsilon} = -(n-1)\phi_{\beta\alpha},$

Inserting equation (3.1) into the equation (3.2), we get

(3.3) $\phi_{\beta\epsilon}R_{\alpha}^{\epsilon} = -(n-1)\phi_{\beta\alpha},$

Contracting equation (3.3) with ϕ_{γ}^{β} and using equation (1.13) yields

(3.4) $R_{\gamma\alpha} = -(n-1)g_{\gamma\alpha},$

Contracting equation (3.4) with $g^{\gamma\alpha}$ yields

(3.5) $R = -n(n-1).$

Consequently, we have a theorem:

Theorem 3.1:

If the P-Sasakian manifold is Ricci symmetric then scalar curvature to be constant.

4. PROJECTIVE SYMMETRIC P-SASAKIAN MANIFOLD

Definition 4.1:

If the projective tensor $P_{\gamma\beta\alpha}^{\epsilon}$ satisfies the condition

(4.1) $\nabla_{\lambda}P_{\gamma\beta\alpha}^{\epsilon} = 0,$

Wherein

(4.2) $P_{\gamma\beta\alpha}^{\epsilon} = R_{\gamma\beta\alpha}^{\epsilon} - \frac{1}{n-1}(R_{\beta\alpha}\delta_{\gamma}^{\epsilon} - R_{\gamma\alpha}\delta_{\beta}^{\epsilon})$

then n-dimensional Riemannian manifold is termed as projective symmetric P-Sasakian manifold [3].

In this regard, we have a theorem:

Theorem 4.1:

In a projective symmetric P-Sasakian manifold, the relation $P_{\rho\gamma\beta\alpha} + P_{\rho\beta\alpha\gamma} + P_{\rho\alpha\gamma\beta} = 0$ holds good.

Proof:

On taking cyclic permutation of the indices γ, β, α and using equation (1.17), we obtain

(4.3) $$P^{\epsilon}_{\gamma\beta\alpha} + P^{\epsilon}_{\beta\alpha\gamma} + P^{\epsilon}_{\alpha\gamma\beta} = -\frac{1}{n-1}[(R_{\beta\alpha}\delta^{\epsilon}_{\gamma} - R_{\gamma\alpha}\delta^{\epsilon}_{\beta})(R_{\alpha\gamma}\delta^{\epsilon}_{\beta} - R_{\beta\gamma}\delta^{\epsilon}_{\alpha})(R_{\gamma\beta}\delta^{\epsilon}_{\alpha} - R_{\alpha\beta}\delta^{\epsilon}_{\gamma})],$$

If the Ricci tensor $R_{\alpha\beta}$ is symmetric then the equation (4.3) becomes

(4.4) $$P^{\epsilon}_{\gamma\beta\alpha} + P^{\epsilon}_{\beta\alpha\gamma} + P^{\epsilon}_{\alpha\gamma\beta} = 0,$$

Contracting equation (4.4) with $g_{\epsilon\rho}$, we obtain

(4.5) $$P_{\rho\gamma\beta\alpha} + P_{\rho\beta\alpha\gamma} + P_{\rho\alpha\gamma\beta} = 0.$$

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