Development of Mathematical Model and its ANN Validation of Thermoelectric Generator System for its performance enhancement

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ABSTRACT
The experimental data based modeling achieved this all the way through mathematical models for the dependent Pi terms. In such complex fact relating non linear systems it is also intended to develop mathematical models using dimensionless analysis. The yield of this network can be evaluated by comparing mathematical model and experimental data. In the present work identify the independent and dependent variables from Thermoelectric Generator (TEG) system and developed the 3 dimensionless independent Pi terms against power of TEG module as dependent variable. This attempts the relevance of dimensionless analysis to find what parameters are influencing the performance of Thermoelectric Generator system (TEG).

Keywords: Mathematical Modeling, Dimensionless analysis, Thermo Electric Generator module, ANN.

1. INTRODUCTION TO TEG

The basic theory and operation of thermoelectric based systems have been developed for many years. Thermoelectric power generation is based on a phenomenon called Seebeck effect discovered by Thomas Seebeck in 1821 [5]. When a temperature difference is established between the hot and cold junctions of two Dissimilar materials (metals or semiconductors) a voltage is generated, i.e., Seebeck voltage. In fact, this phenomenon is applied to thermocouples that are extensively used for temperature measurement.

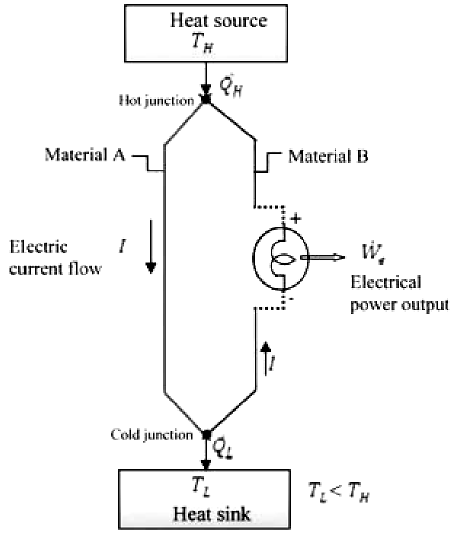


Fig.1. Schematic diagram showing the basic concept of a simple thermoelectric power generator operating based on Seebeck effect. [5]

In above figure heat is transferred at the rate of QH from a high temp. Heat source maintained at TH to the hot junction, and rejected heat rate of QL to low temp. sink maintained at TL. Due to the heat supplied at hot junction, causes the electric current to flow in the circuit to produced electrical voltage [5]

2. EXPERIMENTAL SET UP FOR INVESTIGATING THE PERFORMANCE OF TEG MODULE

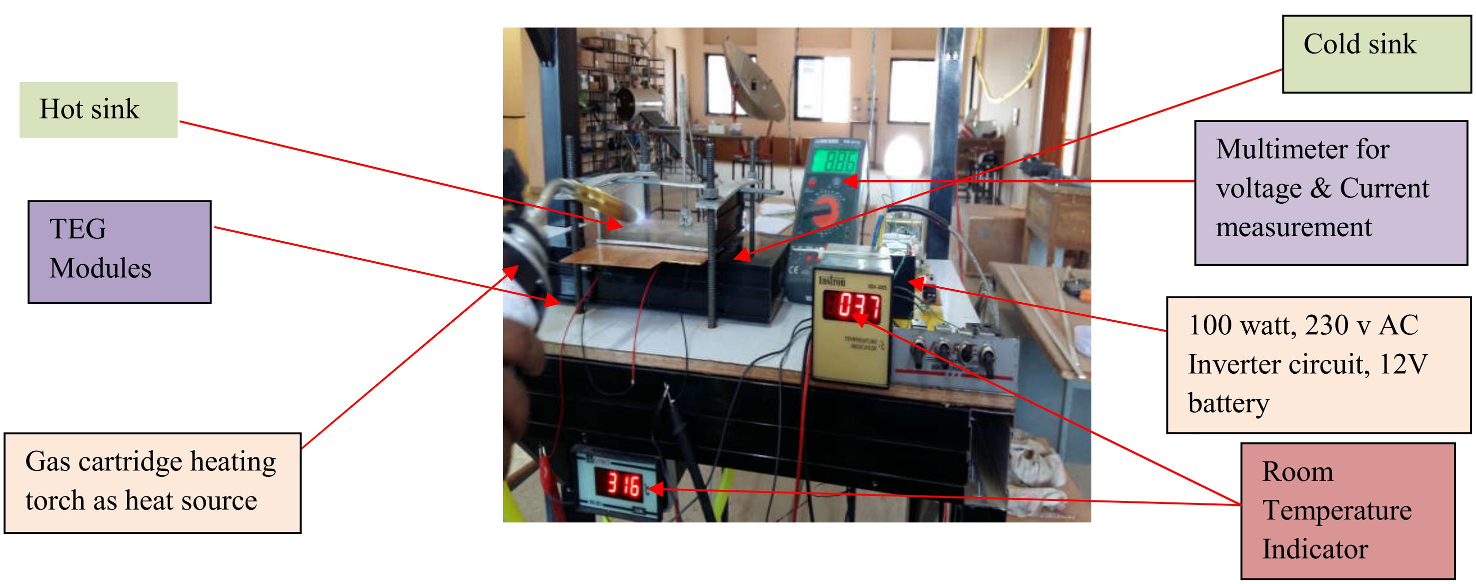


Fig. 2 Picture of TEG experimental setup with Gas cartridge heating torch as heat source [11]

WORKING

Fig. 2 shows TEG experimental setup with Gas cartridge heating torch as heat source is developed and tested [11]. TEG module is placed into a system, whereby the hot side has a higher temperature than the cold side, DC power will be produced. The greater the ΔT (difference in temperature across the module the greater the power produced). During testing hot side temperature is maintained at 300 °C by using Gas cartridge heating torch as heat source and cold side temperature at 30 °C by cold water as source [5]. TEG module terminal connected to multimeter for measurement of voltage and current. and these modules can be placed in parallel and series to produce a workable larger voltage. After testing results were obtained as power output of both TEG modules (TEG Module 1268-4.3 and TEG Module 4199-5.3) is measured 5.0925 W and 6.256 W respectively [11]

3. Identification of Independent and Dependent Variables

3.1 Independent Variables in the form of LMTθ

3.1.1 Variable related to Butane gas portable gas cartridge used as heat source

Pi Term	Code	Name of Independent Variables	LMTΘ	Type of Variables	Dimensionless Pi Term
π1	A1	Mass of butane (m) (Kg)	$L^0M^1T^0\theta^0$	Independent	$\pi1 = \frac{A6\ x\ A9\ x\ A19}{(A7\ x\ A8)}\ x\ \frac{A2}{(A12\ x\ A13\ x\ A14)}\ x\ \frac{A16\ x\ A15}{(A17\ x\ A18)}\ x\ \frac{A4\ x\ A12\ x\ A13\ xA14}{(A1\ )}\ x\ \frac{(A3\ x\ A2)}{A11}\ x\ \frac{(A5\ x\ A5\ x\ A1)}{(A11\ x\ A14\ x\ A15)}$
	A2	Volume of butane gas (V) (m3)	$L^3M^0T^0\theta^0$	Independent	
	A3	Pressure of butane gas (Pb) (N/m2) (Kg/mS2)	$L^{-1}M^1T^{-2}\theta^0$	Independent	
	A4	Density of butane gas (ρ_b) (kg/m^3)	$L^{-3}M^1T^0\theta^0$	Independent	
	A5	Kinematic viscosity of butane gas (m^2/s)	$L^2M^0T^{-1}\theta^0$	Independent	
	A6	Sp heat at constant pressure (Cp) (J/KgK)	$L^2M^0T^{-2}\theta^{-1}$	Independent	
	A7	Sp. heat at constant volume (Cv) (J/KgK)	$L^2M^0T^{-2}\theta^{-1}$	Independent	
	A8	Enthalpy (h) (J/Kg)	$L^2M^0T^{-2}\theta^0$	Independent	
	A9	Entropy (s) (j/Kg-K)	$L^2M^0T^{-2}\theta^{-1}$	Independent	

	A10	Thermal conductivity (W/mK)	$M^1 L^1 T^{-3} \theta^{-1}$	Independent	$x \frac{(A10\ x\ A19\ x\ A20 x\ A12\ )}{(A8\ x\ A1\ )}$
	A11	Total Enthalpy (H) (Joule) (N-m) (Kgm^2/S^2)	$M^1 L^2 T^{-2} \theta^0$	Independent	
	A12	Height of gas cartridge (He) (m)	$L^1 M^0 T^0 \theta^0$	Independent	
	A13	Diameter of gas cartridge (D1) (m)	$L^1 M^0 T^0 \theta^0$	Independent	
	A14	Length of pipe attached to gas cartridge (L1) (m)	$L^1 M^0 T^0 \theta^0$	Independent	
	A15	OD of pipe attached to gas cartridge (D2) (m)	$L^1 M^0 T^0 \theta^0$	Independent	
	A16	Length of nozzle (L2) (m)	$L^1 M^0 T^0 \theta^0$	Independent	
	A17	Tip diameter of nozzle (D3) (m)	$L^1 M^0 T^0 \theta^0$	Independent	
	A18	ID of pipe attached to gas cartridge (D4) (m)	$L^1 M^0 T^0 \theta^0$	Independent	
	A19	Temp of gas inside (θ1) (0K)	$L^0 M^0 T^0 \theta^1$	Independent	
	A20	Time (t1) (s)	$L^0 M^0 T^1 \theta^0$	Independent	

Table: 1 Variables in the form of LMTθ [12]

4. PROCEDURE FOR FORMULATING THE MATHEMATICAL MODEL BY DIMENSIONLESS ANALYSIS TECHNIQUE [12]

Dimensional analysis involves a dimensional model analysis of acting quantities in the investigated process. It enables one to determine, in a simple algebraic way, dimensionless similarity criteria and functional relations, represented amongst them by a criterion equation. Further, it enables the conversion of physical quantities into other various fundamental sets of measuring units, the conversion of measuring units and other procedures. In modeling and experiment, its main function is to reduce the amount of independent variables, to simplify the solution and to generalize the results thereof. It can become an effective method, especially if a complete mathematical model of the investigated process is not known. This is a method, simple from the practical point of view, which does not enable either solving a problem completely or revealing important inner couplings of an investigated phenomenon. However, it is an extraordinarily effective means of obtaining an idea about the behavior of a phenomenon if neither its complete mathematical nor physical descriptions are known. Usually, it is an important physical tool in every more complicated physical, scientific or industrial experiment. The main functions of dimensional analysis are the following.

1. Determination of the number and form of dimensionless quantities which represent the similarity criteria.
2. Reduction of the numbered independent variables in an experiment, simplification of the solution and generalization of its results.
3. Conversion of the basic set of units of the measurement.
4. Conversion of physical quantities into another basic set of units of measurement.
5. Determination of functional relations in cases where the solver does not know more detailed information of the physical principle of the investigated phenomenon and no complete mathematical description of the phenomenon is known.

In application of dimensional analysis, the highest efficiency is reached in its combination with general physical ideas obtained by a solver directly from experiments. The depth of previous knowledge of the physical principles of the investigated phenomenon can influence and extend considerably the possibilities of the dimensional analysis.

5. MODEL FORMULATION

INTRODUCTION

The data of the independent and dependent parameters of the process has been gathered during the experimentation. In this case there are three independent and four dependent pi terms. It is necessary to correlate quantitatively various independent and dependent pi terms involved in the process. This correlation is nothing but a mathematical model as a tool for every process. The optimum values of the independent pi terms can be decided by optimization of these models for maximum output. This section describes the procedure for development of experimental data based mathematical models [12]

5.1 DEVELOPMENT OF EXPERIMENTAL DATABASED MATHEMATICAL MODEL [13]

Three independent pi terms (viz. Π1, Π2 and Π3) and four dependent pi terms (viz. ΠD1, ΠD2, ΠD3 and ΠD4) have been identified in this formulation. Each pi term is assumed to be the function of the available independent pi terms.

Independent pi terms = (Π1, Π2, Π3)

Dependent pi terms = (ΠD1, ΠD2, ΠD3, ΠD4)

Each dependent pi is assumed to be function of the available independent pi terms,

ΠD1, First dependent pi term = f (Π1, Π2, Π3)

ΠD2, Second dependent pi term = f (Π1, Π2, Π3)

ΠD3, Third dependent pi term = f (Π1, Π2, Π3)

ΠD4, Third dependent pi term = f (Π1, Π2, Π3)

Where, "f = function of". A probable exact mathematical form for this phenomenon could be the empirical relationships in between dependent dimensionless ratio and independent dimensionless ratio are assumed to be exponential.

5.2. DEVELOPMENT OF MODEL FOR POWER OUTPUT DEPENDENT PI TERM (ΠD3)

For the dependent pi term ΠD3, we have,

ΠD3 = f (Π1, Π2, Π3)

Where 'f' stands for "function of" and A probable exact mathematical form for this phenomenon could be

$\Pi D3 = k3 \times (\Pi 1)^{a3} \times (\Pi 2)^{b3} \times (\Pi 3)^{c3}$ ------------ (1)

There are four unknown terms in the equation (1), viz. constant of proportionality K3 and indices a3, b3, c3. It is decided to solve this problem by curve fitting technique (Spiegel 1980). To follow this method it is necessary to have the equation in the form as under.

$Z = a + b \times x + c \times y + d \times z + \ldots\ldots$ ------------- (2)

The equation (1) can be brought in the form of equation (2) by taking the log of both sides of the equation (1). By taking the log of both the sides of theses equation

$\text{Log } \Pi D3 = \log [k3 \times \Pi 1^{a3} \times \Pi 2^{b3} \times \Pi 3^{c3}]$

$\text{Log } \Pi D3 = \log k3 + \log \Pi 1^{a3} + \log \Pi 2^{b3} + \log \Pi 3^{c3}$

$\text{Log } \Pi D3 = \log k3 + a3 \log \Pi 1 + b3 \log \Pi 2 + c3 \log \Pi 3$ --------- (3)

Let, log ΠD3 = Z3, Log K3= K3, Log Π1=A, Log Π2=B, Log Π3=C

Then equation (2) can be written as

$Z3 = K3 + a3 \times A + b3 \times B + c3 \times C$ ----------- (4)

Equation (4) is a regression equation of Z3 on A, B and C in a 'n' dimensional coordinate system this represents the regression hyper plane. To determine the regression hyper plane we determine a3, b3 and c3 in equation (4) by multiplying coefficients of a3, b3, and c3, individually. Multiply by A,

$A \times Z3 = A \times K3 + a3 \times A3 + b3 \times AB + c3 \times AC$

Multiply by B

$B \times Z3 = B \times K3 + a3 \times AB + b2 \times B^2 + c3 \times BC$

Multiply by C

$C \times Z3 = C \times K3 + a3\ AC + b3\ BC + c3\ C^2$

Above set of equations are valid for number of reading taken during experimentation, therefore taking summation of these n values. The equation becomes,

$\Sigma Z3 = n \times K3 + a3 \times \Sigma A + b3 \times \Sigma B + c3 \times \Sigma C$ --------------------- (5)

$\Sigma A \times Z3 = K3 \times \Sigma A + a3 \times \Sigma A^2 + b3 \times \Sigma AB + c3 \times \Sigma AC$ -------- (6)

$\Sigma B \times Z3 = K3 \times \Sigma B + a3 \times \Sigma AB + b3 \times \Sigma B^2 + c3 \times \Sigma BC$ --------- (7)

$\Sigma C \times Z3 = K3 \times \Sigma C + a3 \times \Sigma AC + b3 \times \Sigma BC + c3 \times \Sigma C^2$ ---------- (8)

Where n is the number of sets of the values.

Above equations are called normal equations and are obtained as per the definition. In the above sets of equations the values of the multipliers of K3, a3, b3 and c3 are substituted to compute the values of the unknown's (viz. K3, a3, b3 and c3).

Now Consider, equation (a) as

$\Sigma Z3 = n \times K3 + a3 \times \Sigma A + b3 \times \Sigma B$

$\Sigma Z3 = \Sigma \log \Pi D3 = 50.002$	$\Sigma A = -392.82$
$\Sigma B = \Sigma \log \Pi 2 = 1156.2$	$\Sigma C = \Sigma \log \Pi 3 = -1253.4$

Consider equation (b) as

$\Sigma A \times Z3 = K3 \times \Sigma A + a3 \times \Sigma A^2 + b3 \times \Sigma AB + c3 \times \Sigma AC$

In this equation we know

$\Sigma A \times Z3 = -298.12$ $\Sigma A = -392.82$	$\Sigma A^2 = 2443.48$
$\Sigma AB = -5101$	$\Sigma AC = 5043$

Consider equation as

$\Sigma B \times Z3 = K3 \times \Sigma B + a3 \times \Sigma AB + b3 \times \Sigma B^2 + c3 \times \Sigma BC$

In this equation we know

$\Sigma B \times Z3 = 678.8$ $\Sigma B = 1156.2$	$\Sigma B^2 = 13938$
$\Sigma AB = -5101$	$\Sigma BC = -14622.3$

Consider equation as

$\Sigma C \times Z3 = K3 \times \Sigma C + a3 \times \Sigma AC + b3 \times \Sigma BC + c3 \times \Sigma C^2$

In this equation we know

$\Sigma C \times Z3 = -650$ $\Sigma C = -1253.4$	$\Sigma C^2 = 15742$
$\Sigma AC = 5043$	$\Sigma BC = -14622.37$

Thus, by putting all the values the set of equations can be rewritten as

$50.002 = K3 \times 100 + a3 \times (-392.82) + b3 \times (1156.2) + c3 \times (-1253.4)$

$-298.12 = K3 \times (-392.82) + a3 \times (2443.48) + b3 \times (-5101) + c3 \times (5043)$

$678.8 = K3 \times (566.286) + a3 \times (-5101) + b3 \times (13938) + c3 \times (-14622.3)$

$-650 = K3 \times (-1253.4) + a3 \times (5043) + b3 \times (-14622.37) + c3 \times (15742)$

The above equations can be verified in the matrix form and further values of K3, a3, b3, c3, can be obtained by using matrix analysis.

X3 = inv (W) x P3 ---------- (9)
The matrix method of solving these equations using 'MATLAB' is given below.
W = 4 x4 matrix of the multipliers of K3, a3, b3, and c3
P3 = 4 x 1 matrix of the terms on L H S and
X3 =4 x 1 matrix of solutions of values of K3, a3, b3, and c3
Then, the matrix obtained is given by matrix, P3 = W3 x X3

$$Z_1 \quad x \begin{bmatrix} 1 \\ A \\ B \\ C \end{bmatrix} = \begin{bmatrix} n & A & B & C \\ A & A^2 & BA & CA \\ B & AB & B^2 & CB \\ C & AC & BC & C^2 \end{bmatrix} x \begin{bmatrix} K3 \\ a_3 \\ b_3 \\ c_3 \end{bmatrix}$$

In matrix form the equation can be written as

50.002	100	-392.82	1156.2	-1253.4	K3
-298.12	-392.82	2443.48	-5101	5043	a3
678.8	566.286	-5101	13938	-14622.3	b3
-650	-1253.4	5043	-14622.37	15742	c3

Then, following equation type,
[P3] = [W3] [X3]
Gives the unique values of K3, a3, b3 and c3 and antilog of K3 will be the solution for the equation. Using Mat lab, X3= W3\ P3, after solving X3 matrix with K3 and indices a3, b3, c3, are as follows,

K3	2.41262
a3	0.002008
b3	0.219803
c3	0.162544

Hence the model for dependent term pi term $\boldsymbol{\pi_{D3}}$ is
Model: $(\boldsymbol{\pi_{D3}}) = \text{K3}.\{(\boldsymbol{\pi_1})^{\mathbf{a3}}.(\boldsymbol{\pi_2})^{\mathbf{b3}}.(\boldsymbol{\pi_3})^{\mathbf{c3}}\}$

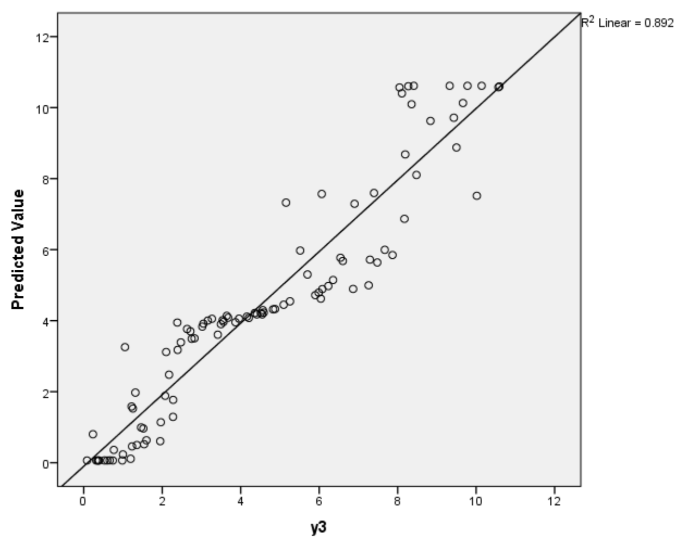
$$(\boldsymbol{\pi_{D3}}) = \mathbf{2.41262}\{(\boldsymbol{\pi_1})^{\mathbf{0.002008}}.(\boldsymbol{\pi_2})^{\mathbf{0.219803}}.(\boldsymbol{\pi_3})^{\mathbf{0.162544}}\}$$

5.2.1. Discussion on Performance of the Models by Analysis of Indices of the Models

The value of curve fitting constant in the model for Power output as dependent Pi term (ΠD3) is 2.41262. This collectively represents the combined effect of all variables related with heat source, heat sink and specifications of TEG module used. Further, as it is positive, this indicates that these causes have strong influence on the dependent process parameters of the Power (ΠD3).The absolute index of π2 is the highest Viz. 0.219803. Thus, this is term related to specifications of heat sink of TEG which is the most influencing π term in this model. The absolute index of π3 is 0.162544, which is related with specification TEG module. This indicates that pi term π3 is also very much influencing term after π2 in this model. The absolute index of π1 is positive viz. 0.002008. This pi term is related with heat source of TEG module. This positive index indicating that variables related with heat source are directly proportional to power produced.

Thus from these models "Intensity of interaction of inputs on deciding Response" can be predicted. The optimization methodology adopted is unique and rigorously derives the most optimum solution for experimental data available for process occurred in thermo electric generator.

The graphical representation between the actual values of dependent terms and values obtained by model with coefficient of determination obtained by Artificial Neural Network are shown in comparative form as follows.



Graph 1: Comparison between Model and experimental data of ∏D3 (R^2 = 0.892)

CONCLUSION

In this research instantaneous active Power output parameters of thermoelectric generator system have been estimated and dimensionless mathematical model developed and validated using ANN model through appropriate training with the data that obtained from the experimental work. Values obtained from ANN and experimental work is observed to be very close to each other, as shown by plots. From the results regression coefficient values (R^2) for the power output dependent term is 0.892 which proves that developed mathematical model for power output of Thermoelectric Generator system is validate. Hence this data would be very much functional for further performance enhancement of Thermoelectric Generator system.

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