MHDDouble Diffusive Rotating Flow Over a Porous Plate: A FDM and FEM Correlative Approach

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Abstract

In the present study, a correlative approach between finite difference and finite element method is made on hydro-magnetic rotating double diffusive flow over a porous vertical plate. The plate is impulsively emerged. The numerical solution of the dimensionless equations is obtained applying Crank-Nicholson and Galerkin finite element methods. Graphical interpretation for momentum, Energy and mass transfer is made and analyzed under different parametric conditions. The numerical results almost tallied. These results showed that finite element method is more efficient.

Keywords
Soret and Dufour; Magneto Hydrodynamics; Porous; Crank-Nicholson and Gelerkin finite element methods.

1 Introduction

Several investigators have focused their interest on porous medium because of their applications in engineering science viz to know the ground level water resources in agricultural field, movement of water in soils; unwanted is removed from the water and what is unwanted from getting into the water system in chemical engineering. Due to these applications, a series of investigations have been made by researchers [1-4]. Double-diffusive convection plays a vital role in mantle convection (magma chambers) and in some scientific applications. Bakr and Riazahb [5] studied the double-diffusive flow past a semi-infinite moving porous vertical plate embedded in a porous medium with chemical reaction and heat source. Mohamed [6] analyzed the influence of on radiative and chemically reacting fluid flow past a vertical moving porous plate with heat source. By making use of modified Darcy–Brinkman model, Dufour and Soret influences on the convection and cross-diffusion
double-diffusive in a Maxwell type fluid in a horizontal layer in permeable media is examined by Awad et al [7]. Hayat et al [8] analyzed Soret and Dufour’s effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid. Incredible contributions related to rotating fluid and mass transfer are reported by many researchers [9-13]. Cogley et al [14] discussed Differential Approximation for Radiation transfer in a non-gray near equilibrium. Hetnarski [15] found an algorithm for generating some inverse Laplace transform of exponential form. Mishra et al [16] studied the Mass transfer effect on MHD flow of viscoelastic fluid through porous medium with oscillatory suction and heat source. Th mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source is studied by Das [17]. Gireesh kumar and Satyanarayana [18] studied the Mass transfer effect on MHD unsteady free convective Walters memory flow with constant suction and heat sink. Ram Prakash Sharma et al [19] studied rotational Impact on Unsteady MHD Double Diffusive Boundary Layer Flow over an Impulsively Emerged Vertical Porous Plate More recently, Radiation and mass transfer effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature is analyzed by Jyotsna Rani Pattnaik et al [20]. In the present paper a correlative approach between implicit finite difference scheme and finite element method is made on double diffusive flow over an impulsively emerged vertical porous plate in fluctuating mass diffusion and temperature.

2 Formulation of the Problem

Flow of an incompressible electrically conducting viscous fluid past a vertical porous plate with variable temperature and mass transfer is considered. The plate is taken as impulsively started. Due to the imposed uniform magnetic field presence, the plate as well as fluid rotates as a rigid body by means of an identical angular velocity \( \Omega^* \). \( y^* \) which makes a ‘\( \pi/2 \)’ angle with the plate. At first, the fluid temperature and concentration close to the plate are supposed to be \( T_\infty^* \) and \( C_\infty^* \) respectively. With a velocity \( u^* = u_0 \), in its own plane at \( t^* > 0 \) the plate begins moving and the concentration and temperature levels in the vicinity of the plate are linearly enhanced with respect to time. Since the plate occupying the plane \( y^* = 0 \) is of infinite extent, on \( y^* \) and \( t^* \) all the quantities depended. It is supposed that the induced
magnetic field is insignificant in order that \( \vec{B} = (0,0,B_0) \). According to the above deductions and from the results of Rajput [9], in non-dimensional form governing equations are given

\[
\frac{\partial u}{\partial t} - 2\Omega v = Gr\theta + Gm\phi + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K} \tag{1}
\]

\[
\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial y^2} - Mv \tag{2}
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{N}{Pr} \theta + Du \frac{\partial^2 \phi}{\partial y^2} + Ec \left( \frac{\partial u}{\partial y} \right)^2 + S\theta \tag{3}
\]

\[
\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + So \frac{\partial^2 \theta}{\partial y^2} - Ch\phi \tag{4}
\]

with corresponding conditions

\( t \leq 0; u = 0, \theta = 0, \phi = 0 \) for all the values of \( y \) \tag{5}

\( t > 0; u = 1, \theta = t, v = 0, \phi = t \) at \( y = 0 \)

\( u \rightarrow 0, \theta \rightarrow 0, v = 0, \phi \rightarrow 0 \) as \( y \rightarrow \infty \)

Where

\[
u = \frac{u'}{u_0}, \quad \nu = \frac{v'}{v_0}, \quad t = \frac{t' u_0^2}{v}, \quad y = \frac{y' u_0}{v}, \quad \phi = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)}, \quad T = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}
\]

\[
Gr = \frac{g\beta v(T'_w - T'_\infty)}{u_0^2}, \quad Gm = \frac{g\beta^* v(C'_w - C'_\infty)}{u_0^2}, \quad \Omega = \frac{\Omega' v}{u_0^2}, \quad So = \frac{D_M K_T (T'_w - T'_\infty)}{v T_M (C'_w - C'_\infty)}
\]

\[
M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \quad Sc = \frac{v}{D}, \quad Pr = \frac{\mu C_p}{k}, \quad N = \frac{16\alpha^* v^2 \sigma T'_\infty}{ku_0^2}, \quad S = \frac{S' v}{\rho C_p u_0^2}
\]

\[
Du = \frac{D_m k_T (C'_w - C'_\infty)}{v C_s C_p (T'_w - T'_\infty)}, \quad Ec = \frac{U_0^2}{C_p (T'_w - T'_\infty)}, \quad Ch = \frac{k_T^2 v}{U_0^2}, \quad T'^4 \approx 4T'^3 T' - 3T'^4 \tag{6}
\]

Here the fluid is non-scattering medium or absorbing, and a gray and emitting radiation a. optically thin gray gas, and the local gradient [21] is given by

\[
\frac{\partial q_r}{\partial y} = -4\alpha^* \sigma (T'^4 - T'^4) \tag{7}
\]
3. Method of solution

(a) Finite element Method

Getting an exact solution for the equations (1)-(4) is very complex. Hence, by making use of Galerkin finite element method, they are solved. Taking linear element for \((y)\), the over two noded \((e)\), \((y, y \xi, y_k)\) is

\[
\frac{\gamma}{2} \left\{ \phi^{(e)}(y) \left[ \frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} - Mu^{(e)} + R_1 \right] \right\} dy = 0
\]  

(8)

Here \( R_1 = Gr\theta + GcC + N, N = \frac{1}{K} \)

Integrating the first term in equation (7), using by-parts method and ignoring that term. After that, put back finite element approximation over the two hugged linear variable '(e)' of the form:

\[
u^{(e)} = N^{(e)} \xi^{(e)}, \text{here } \xi^{(e)} = \left[ \zeta_j \ zeta_k \right], \phi^{(e)} = \left[ u_j \ u_k \right]^T, \zeta_j = \frac{y_k - y_j}{h}, \zeta_k = \frac{y_j - y_k}{h}, h = y_k - y_j
\]

are the basis functions along the \( j^{th} \) and \( k^{th} \) nodes, velocity components \( u_j, u_k \). The consecutive element equations \((y_{i-1}, y_i)\) and \((y_i, y_{i+1})\), adding the element equations by inter-element connectivity, we get

\[
\frac{1}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{\gamma \epsilon^2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{M}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{R_1}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}
\]  

(9)

The following are obtained, after Applying Crank Nicholson method on (1) to (4).

\[
A_i u_{i+1}^{j+1} + A_i u_{i-1}^{j+1} + A_i u_{i+1}^j + A_i u_{i+1}^j + A_i u_{i-1}^j + A_i u_{i+1}^j + R^*
\]  

(10)

\[
D_j v_{i+1}^{j+1} + D_j v_{i-1}^{j+1} + D_j v_{i+1}^j + D_j v_{i+1}^j + D_j v_{i+1}^j + D_j v_{i+1}^j + R^*
\]  

(11)

\[
G_j \theta_i^{j+1} + G_j \theta_{i+1}^{j+1} + G_j \theta_i^j + G_j \theta_{i+1}^j + G_j \theta_i^j + G_j \theta_{i+1}^j + R^***
\]  

(12)

\[
H_i C_i^{j+1} + H_i C_{i+1}^{j+1} + H_i C_i^j + H_i C_{i+1}^j + H_i C_i^j + H_i C_{i+1}^j + R^{****}
\]  

(13)
Here Index \( i \) designates to space and \( j \) for time. At every internal nodal point and on a particular \( n \)-level the finite-difference equations represent a tri-diagonal system. Using the Thomas algorithm the above system is solved.

(b) Finite difference Method

Using Crank-Nicholson method, solving equations (1) to (4), we obtain

\[
-\frac{r}{2} u_{i-1}^{j+1} + (1 + r) u_i^{j+1} - \frac{r}{2} u_{i+1}^{j+1} = a_i^j
\]  \hspace{1cm} (15)

\[
-\frac{r}{2} v_{i-1}^{j+1} + (1 + r) v_i^{j+1} - \frac{r}{2} v_{i+1}^{j+1} = b_i^j
\]  \hspace{1cm} (16)

\[
-\frac{r}{2Pr} \theta_{i-1}^{j+1} + (1 + \frac{r}{Pr}) \theta_i^{j+1} - \frac{r}{2Pr} \theta_{i+1}^{j+1} = d_i^j
\]  \hspace{1cm} (17)

\[
-\frac{r}{2Sc} \phi_{i-1}^{j+1} + (1 + \frac{r}{Sc}) \phi_i^{j+1} - \frac{r}{2Sc} \phi_{i+1}^{j+1} = e_i^j
\]  \hspace{1cm} (18)

with finite difference form conditions

\[
u(i,j) = 0, \theta(i,j) = 0, \phi(i,j) = 1 \text{ for all } i, j
\]

\[
u(0,j) = 0, \theta(0,j) = j\Delta t, \phi(i,j) = j\Delta t \ \forall j
\]  \hspace{1cm} (19)

\[
u(\infty,j) \to 0, \theta(\infty,j) \to 0, \phi(\infty,j) \to 0 \ \forall j
\]
Where,

\[
\frac{\partial f}{\partial t} = \frac{f_{i+1}^j - f_i^j}{\Delta t}, \quad \frac{\partial f}{\partial y} = \frac{f_{i+1}^{j+1} - f_i^{j+1}}{\Delta y}, \quad \frac{\partial^2 f}{\partial y^2} = \frac{1}{2} \left( \frac{f_{i-1}^{j+1} - 2f_i^{j+1} + f_{i+1}^{j+1}}{\Delta y^2} - \frac{f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1}}{(\Delta y)^2} \right)
\]

\[f\] stands \(u, \theta, v, \phi,\] and \(r = \frac{\Delta t}{(\Delta y)^2}, \phi = C\)

\[a_i^j = \frac{r}{2} u_{i-1}^j + \left( 1 - r + M \Delta t - \frac{\Delta t}{K} \right) u_i^j + \frac{r}{2} u_{i+1}^j + \Delta t (Gr \theta_i^j + Gm \phi_i^j) + 2 \Omega \Delta t v_i^j\]

\[b_i^j = \frac{r}{2} v_{i-1}^j + \left( 1 - r + M \Delta t \right) v_i^j + \frac{r}{2} v_{i+1}^j - 2 \Omega \Delta t v_i^j\]

\[d_i^j = \frac{r}{2Pr} \theta_{i-1}^j + \left( 1 - \frac{r}{Pr} - \frac{N \Delta t}{Pr} + S \Delta t \right) \theta_i^j + \frac{r}{2Pr} \theta_{i+1}^j + Ec \left( r_{i+1}^j - r_i^j \right)^2 + Du \left[ \phi(i-1) - 2\phi(i) + \phi(i+1) \right]\]

\[e_i^j = \frac{r}{2Sc} \phi_{i-1}^j + \left( 1 - \frac{r}{Sc} - Ch \Delta t \right) \phi_i^j + \frac{r}{2Sc} \phi_{i+1}^j + So \left[ \theta(i-1) - 2\theta(i) + \theta(i+1) \right]\]

Here mesh sizes along \(y\) and time \(t\) direction are \(\Delta y\) and \(\Delta t\) respectively. space shows \(i\) and time for \(j\)

Fig 4.3.1: Finite-difference scheme presentation as a Grid meshing
To obtain the finite difference equations, the flow region is divided into a mesh of lines parallel to $y$ and $t$ axes as shown in the figure. The finite-difference equations at each inside nodal point on a meticulous $n$-level represent a tri-diagonal system of equations. Using Thomas algorithm, tri-diagonal system is solved. By testing with various grid sizes, the grid independent test is carried out. In the last stage, a step size ‘$\Delta y = 0.1$’ of grid is selected for which computational time and the round-off error gets minimized. So the computation is carried out by taking $\Delta y = 0.1$ and $\Delta t = 0.005$ and the iterations performed, a tolerance $10^{-8}$ is attained. Finite difference scheme is convergent and stable as there is no major variation is observed in the values of $u, v, \theta$ and $\phi$.

4 RESULTS AND DISCUSSION:

Inherent physics of the problem of investigation is analyzed with a suitable CFD tools namely, Finite difference and Finite element techniques. Impact of different physical parameters of the flow involved is investigated by taking the consideration of their graphical representations.

Figures (1) & (9) show that as the value of Eckert number (Ec) enhances fluid velocity and temperature increase. As growing values of Ec within in the system a viscous dissipative heat is developed, consequently, there is rise in the fluid temperature and velocity for the growing values of Ec. Figures (2) and (8) reveals that in the presence magnetic parameter the main flow velocity reduces as Lorentz resistive type body force suppress the flow thus reduces the primary velocity. But in the case of secondary flow velocity a reverse effect is noted as the Lorentz resulting force works as a supporting body force on the secondary velocity flow pattern.

Figures (3) and (7) depicts that primary velocity declines where as secondary flow velocity enhances for growing values of $\Omega$, as fluid motion is reduced by the rotation in the main flow direction but the reverse effect on the secondary flow is noted. This because of the reality that a force (Coriolis) works as a constraint in the direction of primary flow which causes an enhance in the cross flow velocity when at the moving plate the frictional layer is put into the motion. Figures (4) and (10) show that as the value of thermal radiation increases that fluid velocity and temperature falls down. Because the rate of radiative heat, moved to
the fluid is reduced by the increasing values of radiation. Consequently fluid particles kinetic energy is decelerated.

Figures (5) and (12) show that there is fall in the fluid velocity and temperature when Soret number ‘So’ value enhances. As the viscosity of the fluid is reduced by the increasing values of So. So Effects of inertial forces are strengthened and viscous is weakened. Accordingly the fluid temperature and velocity increase. Figures (6) and (11) shows that increasing values of Dufour number enhances the fluid velocity and temperature as growing Dufuor affects on concentration gradients to thermal energy fluxes. Figure (13) reveals that decrease in the Concentration for the growing Ch values as the contribution with regard to the chemical reaction is always dependent on strength of electrolytes accordingly the absorption or adsorption effects matters. The results obtained are fine conformity with realistic physical phenomenon.

5. Conclusions:

The following conclusions are drawn from the above analysis.

1. Significant improvement in main and a secondary fluid flow velocity is noted as Soret and Dufour parameter increases.
2. The results obtained are tallied with realistic physical phenomenon.
3. Correlation study of FDM and FEM for given non-linear fluid dynamics problem revealed that FEM is more efficient because it covers the each and every element of boundary layer from which the grid is constituted. This constraint is not considered for any other scheme.
### 6 NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$C'_\infty$</td>
<td>Concentration in the fluid far away from the plate</td>
</tr>
<tr>
<td>$C'_W$</td>
<td>Concentration of the fluid in the vicinity of the plate</td>
</tr>
<tr>
<td>$T'_\infty$</td>
<td>Temperature of the fluid far away from the plate</td>
</tr>
<tr>
<td>$T'_W$</td>
<td>Constant temperature of the plate</td>
</tr>
<tr>
<td>$S'$</td>
<td>Constant heat source with dimension</td>
</tr>
<tr>
<td>$t'$</td>
<td>time with dimension</td>
</tr>
<tr>
<td>$t$</td>
<td>time (Dimensionless)</td>
</tr>
<tr>
<td>$D_M$</td>
<td>Coefficient of mass diffusivity</td>
</tr>
<tr>
<td>$T'$</td>
<td>Temperature of the plate</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Thermal diffusion ratio</td>
</tr>
<tr>
<td>$T_M$</td>
<td>The mean fluid temperature</td>
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<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure,</td>
</tr>
<tr>
<td>$u'$</td>
<td>Primary velocity with dimension</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number</td>
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<tr>
<td>$u$</td>
<td>Dimensionless velocity along x-axis</td>
</tr>
<tr>
<td>Gm</td>
<td>Modified Grashof number</td>
</tr>
<tr>
<td>$a^*$</td>
<td>Absorption coefficient</td>
</tr>
<tr>
<td>$v'$</td>
<td>Secondary velocity of the fluid</td>
</tr>
<tr>
<td>M</td>
<td>Magnetic parameter</td>
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<tr>
<td>$v$</td>
<td>Dimensionless velocity along y-axis</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Radiative heat flux</td>
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<tr>
<td>$y'$</td>
<td>Coordinate axis normal to the plate</td>
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<tr>
<td>Sc</td>
<td>Schmidt number</td>
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<tr>
<td>Pr</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>Volumetric coefficient of expansion for concentration</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Volumetric coefficient of thermal expansion</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Rotation parameter</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant</td>
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<tr>
<td>$k$</td>
<td>Thermal conductivity of the fluid</td>
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<td>$\rho$</td>
<td>Density of the fluid</td>
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<td>$\nu$</td>
<td>Kinematic viscosity</td>
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<td>Dimensionless rotational parameter</td>
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<td>$\mu$</td>
<td>Coefficient of viscosity</td>
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<td>N</td>
<td>Radiative parameter</td>
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<tr>
<td>$B_0$</td>
<td>Magnetic field strength</td>
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<tr>
<td>S</td>
<td>Heat source/sink parameter</td>
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<tr>
<td>Kr or Ch</td>
<td>Chemical reaction parameter</td>
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</table>
Graphs using FDM

Graphs using FEM

**Fig 1:** Primary velocity field
(t = 2; Gm=5; Gr=5.0; K=0.1; N=1.0; S=1.0; Sc=0.22; Ω=0.5; Pr=0.71; Ch=0.5; Du=1.0; So=1.0)

**Fig 2:** Primary velocity field
(t = 2; Gm=5; Gr=5.0; K=0.1; Ec=1.0; N=1.0; S=1.0; Sc=0.22; Ω=0.5; Pr=0.71; Ch=0.5; Du=1.0; So=1.0)

**Fig 3:** Primary velocity field
(t = 2; Gm=5; Gr=5.0; K=0.1; Ec=1.0; N=1.0; M=1.0; Sc=0.22; Ω=0.5; Pr=0.71; Ch=0.5; Du=1.0; So=1.0)
Fig 4: Primary velocity field
(t=2; Gm=5; G[r]=5.0; K=0.1, S=1.0; Ec=1.0; N=1.0; Sc=0.22; ω=0.5; Pr=0.71; Ch=0.5; Du=1.0, So=1.0)

Fig 5: Primary velocity field
(t=2; Gm=5; G[r]=5.0; K=0.1, S=1.0; N=1.0; M=1.0; Ec=1.0; So=2.0; S=1.0; N=1.0; Sc=0.22; ω=0.5; Pr=0.71; Ch=0.5; Du=1.0)

Fig 6: Primary velocity field
(t=1.0; Gm=5; G[r]=5.0; K=0.1, M=1.0; Ec=1.0; N=1.0; Ch=0.5; S=1.0; S=1.0; So=1.0; M=1.0; Ec=1.0; N=1.0; Sc=0.22; ω=0.5; Pr=0.71; Ch=0.5; Du=1.0)
Fig 7: Secondary velocity field
(t = 1.0; Gm = 5; Gr = 5.0; K = 0.1; M = 1.0; Ec = 1.0; N = 1.0; Ch = 0.5; Sc = 0.22; Pr = 0.71; S = 1.0; Du = 1.0; So = 1.0)

Fig 8: Secondary velocity field
(t = 2.0; Gm = 5; Gr = 5.0; K = 0.1; Ec = 1.0; N = 1.0; Ch = 0.5; Sc = 0.22; Pr = 0.71; S = 1.0; Du = 1.0; So = 1.0)

Fig 9: Temperature field
(t = 1.0; Gm = 5; Gr = 5.0; K = 0.1; M = 1.0; N = 1.0; Ch = 0.5; Sc = 0.22; Pr = 0.71; S = 1.0; Du = 1.0; So = 1.0)
Fig 10: Temperature field
\( t = 2.0; \ G_m = 5; \ G_r = 5.0; \ K = 0.1; \ M = 1.0; \ E_c = 1.0; \ N = 0.1; \ S = 1.0; \ D_u = 1.0; \ S_o = 1.0 \)

Fig 11: Temperature field
\( t = 2.0; \ G_m = 5; \ G_r = 5.0; \ M = 1.0; \ E_c = 1.0; \ N = 1.0; \ C_h = 0.5; \ S = 1.0; \ D_u = 1.0 \)

Fig 12: Concentration field
\( t = 1.0; \ G_m = 5; \ G_r = 5.0; \ M = 1.0; \ E_c = 1.0; \ N = 1.0; \ S_o = 1.0; \ D_u = 1.0 \)
REFERENCES


