On Double Fuzzy Contra $\mathcal{Z}$ Continuous Functions

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Abstract

In this paper we introduce double fuzzy contra $\mathcal{Z}$-continuous functions and study some of their properties in double fuzzy topological spaces.

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1 Introduction

" Intuitionistic fuzzy sets " were first introduced by Atanassov [1] in 1993, then Coker [2] introduced the notion of " Intuitionistic fuzzy topological space " in 1997. In 2005, Garcia and Rodabaugh [5] proved that the term " intuitionistic " is unsuitable in mathematics and applications and they introduced the name double for the term intuitionistic. In the past two decades many researchers [9, 10, 20] doing more applications on double fuzzy topological spaces. From 2011, El-Maghrabi and Mubarki [7] introduced and studied some properties of $\mathcal{Z}$-open sets and maps in topological spaces. Recently These kind of sets and maps were studied in $\hat{S}$ostak's Fuzzy topological spaces [18, 19]. Ekici and Kerre [3] introduced the concept of fuzzy contra continuous functions. In double fuzzy topological spaces, [16, 17] introduced (ι,κ)- fuzzy $\mathcal{Z}$ closed sets and using them double fuzzy $\mathcal{Z}$ continuous functions were studied by Shiventhira devi sathaananthan et. al.

2 Preliminaries

Throughout this paper, X will be a non-empty set, I is the closed unit interval [0,1], $I_0 = (0,1]$, $I_1 = [0,1)$, $\iota \in I_0$, $\kappa \in I_1$ and always $\iota + \kappa \leq 1$. A fuzzy set μ is quasi-coincident with a fuzzy set ν denoted by $\mu q \nu$ iff there exists $x \in X$ such that $\mu(x) + \nu(x) > 1$ and otherwise they are not quasi-coincident which denoted by $\mu \bar{q} \nu$. The family of all fuzzy sets on X (resp. Y and Z) is denoted by I^X (resp. I^Y and I^Z). By $\underline{0}$ and $\underline{1}$, we denote the smallest and the largest fuzzy sets on X. For a fuzzy set $\mu(x) \in I^X$, $\underline{1} - \mu(x)$ denotes its complement. For $x \in X, \iota \in I_0$, a fuzzy point x_ι is defined by $x_r(y) = \iota$ if $x = y$ for all other $y, x_r(y) = 0$. All other notations are standard notations of fuzzy set theory.

Definition 2.1 *[14] A double fuzzy topology* (τ, τ^*) *on* X *is a pair of maps* $\tau, \tau^*: I^X \to I$, *which satisfies the following properties:*

1. $\tau(\lambda) \leq \underline{1} - \tau^*(\lambda)$ for each $\lambda \in I^X$.
2. $\tau(\lambda_1 \wedge \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$.
3. $\tau(\vee_{i\in\Gamma} \lambda_i) \leq \wedge_{i\in\Gamma} \tau(\lambda_i))$ and $\tau^*(\vee_{i\in\Gamma} \lambda_i) \leq \vee_{i\in\Gamma} \tau^*(\lambda_i)$ for each $\lambda_i, \in I^X, i \in \Gamma$.

The triplet (X, τ, τ^*) is called a double fuzzy topological space (briefly, $DFts$). A fuzzy set λ is called an (ι, κ)-fuzzy open (briefly (ι, κ)-fo) set if $\tau(\lambda) \geq \iota$ and $\tau^*(\lambda) \leq \kappa$, λ is called an (ι, κ)-fuzzy closed (briefly (ι, κ)-fc) set iff $\underline{1} - \lambda$ is an (ι, κ)-fo set.

Definition 2.2 *[6] Let (X, τ, τ^*) be a $DFts$. Then double fuzzy interior and double fuzzy closure operators are defined from $I^X \times I_0 \times I_1 \to I^X$ as follows*

$$I_{\tau,\tau^*}(\lambda, \iota, \kappa) = \vee \{\mu \in I^X | \mu \leq \lambda, \tau(\mu) \geq \iota, \tau^*(\mu) \leq \kappa\},$$

$$C_{\tau,\tau^*}(\lambda, \iota, \kappa) = \wedge \{\mu \in I^X | \mu \geq \lambda, \tau(\underline{1} - \mu) \geq \iota, \tau^*(\underline{1} - \mu) \leq \kappa\},$$

where $\iota \in I_0$ and $\kappa \in I_1$ such that $\iota + \kappa \leq 1$.

Definition 2.3 *[13] Let (X, τ, τ^*) be a $DFts$. Then for each $\iota \in I_0$, $\kappa \in I_1$, a fuzzy set $\lambda \in I^X$, is said to be [(i)]*

1. (ι, κ)-fuzzy regular open (briefly (ι, κ)-fro) set if $\lambda = I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$.
2. (ι, κ)-fuzzy regular closed (briefly (ι, κ)-frc) set iff $\underline{1} - \lambda$ is (ι, κ)-fro set.

Definition 2.4 *[11] Let (X, τ, τ^*) be a $DFts$. Then for each $\iota \in I_0$, $\kappa \in I_1$, and for fuzzy set $\lambda \in I^X$, we define the operators $\delta C_{\tau,\tau^*}$ and $\delta I_{\tau,\tau^*}$: $I^X \times I_0 \times I_1 \to I^X$ as follows*

$$\delta I_{\tau,\tau^*}(\lambda, \iota, \kappa) = \vee \{\mu \in I^X | \mu \leq \lambda, \mu \ \text{ is an } \ (\iota, \kappa) - fro\},$$

$$\delta C_{\tau,\tau^*}(\lambda, \iota, \kappa) = \wedge \{\mu \in I^X | \mu \geq \lambda, \mu \ \text{ is an } \ (\iota, \kappa) - frc\}.$$

Definition 2.5 *[4, 7, 11] Let (X, τ, τ^*) be a $DFts$. Then for each $\iota \in I_0$, $\kappa \in I_1$, a fuzzy set $\lambda \in I^X$, is said to be [(i)]*

1. (ι, κ)-fuzzy δ open (briefly (ι, κ)-$f\delta o$) set if $\lambda = \delta I_{\tau,\tau^*}(\lambda, \iota, \kappa)$.
2. (ι, κ)-fuzzy pre open (resp. (ι, κ)-fuzzy semi open) (briefly (ι, κ)-fpo (resp. (ι, κ)-fso)) set if $\lambda \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$ (resp. $\lambda \leq C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$).
3. (ι, κ)-fuzzy δ pre open (resp. (ι, κ)-fuzzy δ semi open, (ι, κ)-fuzzy b open, (ι, κ)-fuzzy $\mathcal{Z}$ open [16] and (ι, κ)-fuzzy e open) (briefly (ι, κ)-$f\delta po$ (resp. (ι, κ)-$f\delta so$, (ι, κ)-fbo, (ι, κ)-$f\mathcal{Z}o$ and (ι, κ)-feo)) set if $\lambda \leq I_{\tau,\tau^*}(\delta C_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$ (resp. $\lambda \leq C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$, $\lambda \leq C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa) \vee I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$, $\lambda \leq C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa) \vee I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$ and $\lambda \leq C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa) \vee I_{\tau,\tau^*}(\delta C_{\tau,\tau^*}(\lambda, \iota, \kappa), \iota, \kappa)$).
4. (ι, κ)-fuzzy δ pre closed (resp. (ι, κ)-fuzzy pre closed, (ι, κ)-fuzzy semi closed, (ι, κ)-fuzzy δ semi closed, (ι, κ)-fuzzy b closed, (ι, κ)-fuzzy $\mathcal{Z}$ closed and (ι, κ)-fuzzy e closed) (briefly (ι, κ)-$f\delta pc$ (resp. (ι, κ)-fpc, (ι, κ)-fsc, (ι, κ)-$f\delta sc$, (ι, κ)-fbc, (ι, κ)-$f\mathcal{Z}c$ and (ι, κ)-fec)) set if $\underline{1} - \lambda$ is an (ι, κ)-$f\delta po$ (resp. (ι, κ)-fpo, (ι, κ)-fso, (ι, κ)-$f\delta so$, (ι, κ)-fbo, (ι, κ)-$f\mathcal{Z}o$ and (ι, κ)-feo).

Definition 2.6 *[11, 16] Let (X, τ, τ^*) be a $DFts$. Then for each $\iota \in I_0$, $\kappa \in I_1$ and for fuzzy set $\lambda \in I^X$, we define the operators $\delta PC_{\tau,\tau^*}$ (resp. $\mathcal{Z}C_{\tau,\tau^*}$ and eC_{τ,τ^*}) and $\delta PI_{\tau,\tau^*}$ (resp. $\mathcal{Z}I_{\tau,\tau^*}$ and eI_{τ,τ^*}): $I^X \times I_0 \times I_1 \to I^X$ as follows*

$$\delta PI_{\tau,\tau^*} \ (\ resp. \ \mathcal{Z}I_{\tau,\tau^*} \ and \ eI_{\tau,\tau^*})(\lambda, \iota, \kappa) = \vee \{\mu \in I^X | \mu \leq \lambda, \mu \ \text{ is an } \ (\iota, \kappa) - f\delta po \ (\ resp. \ (\iota, \kappa) - f\mathcal{Z}o \ and \ (\iota, \kappa) - feo)\},$$

$$\delta PC_{\tau,\tau^*} \ (\ resp. \ \mathcal{Z}C_{\tau,\tau^*} \ and \ eC_{\tau,\tau^*})(\lambda, \iota, \kappa) = \wedge \{\mu \in I^X | \mu \geq \lambda, \mu \ \text{ is an } \ (\iota, \kappa) - f\delta pc \ (\ resp. \ (\iota, \kappa) - f\mathcal{Z}c \ and \ (\iota, \kappa) - fec)\}.$$

Definition 2.7 *[16] Let* (X,τ,τ^*) *be a* $DFts$, $\lambda \in I^X$, $\iota \in I_0$ *and* $\kappa \in I_1$, λ *is called an* (ι,κ)*- fuzzy* $\mathcal{Z}$*-*Q*-neighborhood of* $x_t \in P_t(X)$ *if there exists an* (ι,κ)*-*$f\mathcal{Z}o$ *set* $\mu \in I^X$ *such that* $x_t q \mu$ *and* $\mu \leq \lambda$. *The family of all* (ι,κ)*-fuzzy* $\mathcal{Z}$*-*Q*-neighborhood of* x_t *is denoted by* $\mathcal{Z}Q$-(x_t,ι,κ).

Proposition 2.1 *[16] Let* (X,τ,τ^*) *be a* $DFts$, $\lambda,\mu \in I^X$, *then*

1. $\mathcal{Z}I_{\tau,\tau^*}(\underline{0},\iota,\kappa) = \underline{0}, \mathcal{Z}C_{\tau,\tau^*}(\underline{0},\iota,\kappa) = \underline{0}$ and $\mathcal{Z}I_{\tau,\tau^*}(\underline{1},\iota,\kappa) = \underline{1}, \mathcal{Z}C_{\tau,\tau^*}(\underline{1},\iota,\kappa) = \underline{1}$.
2. $\underline{1} - \mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) = \mathcal{Z}C_{\tau,\tau^*}(\underline{1}-\lambda,\iota,\kappa)$ and $\underline{1} - \mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa) = \mathcal{Z}I_{\tau,\tau^*}(\underline{1}-\lambda,\iota,\kappa)$.
3. If $\lambda < \mu$ then $\mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) < \mathcal{Z}I_{\tau,\tau^*}(\mu,\iota,\kappa)$ and $\mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa) < \mathcal{Z}C_{\tau,\tau^*}(\mu,\iota,\kappa)$.
4. $\mathcal{Z}C_{\tau,\tau^*}(\lambda \vee \mu,\iota,\kappa) > \mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa) \vee \mathcal{Z}C_{\tau,\tau^*}(\mu,\iota,\kappa)$.
5. $\mathcal{Z}I_{\tau,\tau^*}(\lambda \vee \mu,\iota,\kappa) > \mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) \vee \mathcal{Z}I_{\tau,\tau^*}(\mu,\iota,\kappa)$.
6. $\mathcal{Z}C_{\tau,\tau^*}(\lambda \wedge \mu,\iota,\kappa) < \mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa) \wedge \mathcal{Z}C_{\tau,\tau^*}(\mu,\iota,\kappa)$
7. $\mathcal{Z}I_{\tau,\tau^*}(\lambda \wedge \mu,\iota,\kappa) < \mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) \wedge \mathcal{Z}I_{\tau,\tau^*}(\mu,\iota,\kappa)$.

The operators $\delta I_{\tau,\tau^*}(\lambda,\iota,\kappa)$ and $\delta s I_{\tau,\tau^*}(\lambda,\iota,\kappa)$ satisfy the above properties.

Proposition 2.2 *[16] Let* (X,τ,τ^*) *be a* $DFts$, $\lambda,\mu \in I^X$, *then*

1. $\lambda \leq \mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq C_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq \delta C_{\tau,\tau^*}(\lambda,\iota,\kappa)$.
2. $\delta I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq \mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq \lambda$.

Theorem 2.1 *[16] Let* (X,τ,τ^*) *be a* $DFts$, *for each* $\lambda,\mu \in I^X$, *then the operator* (ι,κ)*-*$\mathcal{Z}C_{\tau,\tau^*}$ *satisfies the following statements*

1. $\mathcal{Z}C_{\tau,\tau^*}(\mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa) = \mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa)$.
2. If λ is (ι,κ)-$f\mathcal{Z}c$ set then $\mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa) = \lambda$.
3. If μ is (ι,κ)-$f\mathcal{Z}o$ set then $\mu q \lambda$ iff $\mu q \mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa)$.

Theorem 2.2 *[16] Let* (X,τ,τ^*) *be a* $DFts$, *for each* $\lambda,\mu \in I^X$, *then the operator* (ι,κ)*-*$\mathcal{Z}I_{\tau,\tau^*}$ *satisfies the following statements*

1. $\mathcal{Z}I_{\tau,\tau^*}(\mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa) = \mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa)$.
2. If λ is (ι,κ)-$f\mathcal{Z}o$ set then $\mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) = \lambda$.
3. If $\lambda \leq \mu$ then $\mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq \mathcal{Z}I_{\tau,\tau^*}(\mu,\iota,\kappa)$.
4. $\mathcal{Z}I_{\tau,\tau^*}(\underline{1}-\lambda,\iota,\kappa) = \underline{1} - \mathcal{Z}C_{\tau,\tau^*}(\lambda,\iota,\kappa)$ and $\mathcal{Z}C_{\tau,\tau^*}(\underline{1}-\lambda,\iota,\kappa) = \underline{1} - \mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa)$

Definition 2.8 *[15] A function* f *from a* $DFts$ (X,τ,τ^*) *to a* $DFts$ (Y,σ,σ^*) *is called as double fuzzy continuous (resp. double fuzzy* δ *pre continuous, double fuzzy* δ *semi continuous, double fuzzy semi continuous [8], double fuzzy* $\mathcal{Z}$ *continuous and double fuzzy* e *continuous) (briefly* $DFCts$*, (resp.* $DF\delta pCts$, $DF\delta sCts$, $DFsCts$, $DFMCts$ *and* $DFeCts$*)) function if* $f^{-1}(\mu)$ *is an* (ι,κ)*-*fc *(resp.* (ι,κ)*-*$f\delta pc$, (ι,κ)*-*$f\delta sc$, (ι,κ)*-*fsc, (ι,κ)*-*$f\mathcal{Z}c$ *and* (ι,κ)*-*fec*) set in* I^X *for every* (ι,κ)*-*fc *set* $\mu \in I^Y$ *for all* $\iota \in I_0$ *and* $\kappa \in I_1$.

Definition 2.9 *[15] A fuzzy set* λ *in a* $DFts$ (X,τ,τ^*) *is called an* (ι,κ)*-fuzzy dense (resp.* (ι,κ)*-fuzzy nowhere dense) if there exists no* (ι,κ)*-*fo *(resp. non-zero* (ι,κ)*-*fo*) set* μ *in* (X,τ,τ^*) *such that* $\lambda < \mu < \underline{1}$ *(resp.* $\mu < C_{\tau,\tau^*}(\lambda,\iota,\kappa)$. *)*

Definition 2.10 *[12] A function* f *from a dfts* (X,τ,τ^*) *to a dfts* (Y,σ,σ^*)*. Then* f *is called as double fuzzy contra continuous (resp. double fuzzy contra* δ *pre continuous and double fuzzy contra* e *continuous) (briefly* $DFcCts$*, (resp.* $DFc\delta pCts$ *and* $DFceCts$*)) function if* $f^{-1}(\mu)$ *is an* (ι,κ)*-*fc *(resp.* (ι,κ)*-*$f\delta pc$ *and* (ι,κ)*-*fec*) set in* I^X *for every* (ι,κ)*-*fo *set* $\mu \in I^Y$ *for all* $\iota \in I_0$ *and* $\kappa \in I_1$.

3 A double fuzzy contra $\mathcal{Z}$ Continuous functions

In this section we introduce the class of double fuzzy contra δ(resp. δ semi and $\mathcal{Z}$) continuous functions and discuss about their properties.

Definition 3.1 *A function f from a dfts (X,τ,τ^*) to a dfts (Y,σ,σ^*). Then f is called as double fuzzy contra $\mathcal{Z}$ (resp. δ and δ semi) continuous (briefly $DFc\mathcal{Z}ts$, (resp. $DFc\delta Cts$ and $DFc\delta sCts$)) function if $f^{-1}(\mu)$ is an (ι,κ)-$f\mathcal{Z}c$ (resp. (ι,κ)-$f\delta c$ and (ι,κ)-$f\delta sc$) set in I^X for every (ι,κ)-fo set $\mu\in I^Y$ for all $\iota\in I_0$ and $\kappa\in I_1$.*

Theorem 3.1 *Let $f:(X,\tau,\tau^*)\rightarrow(Y,\eta,\eta^*)$ be a mapping*

1. Every $DFc\delta Cts$ function is $DFcCts$ (resp. $DFc\delta pCts$ and $DFc\delta sCts$) function.
2. Every $DFcCts$ (resp. $DFc\delta sCts$) function is $DFc\mathcal{Z}Cts$ function.
3. Every $DFc\delta pCts$ (resp. $DFc\delta sCts$) function is $DFceCts$ function.
4. Every $DFc\mathcal{Z}Cts$ function is $DFceCts$ function .

Remark 3.1 *The converse of the above theorem, in general, need not be true. It can be verified from the following examples.*

Example 3.1 *Let $X=Y=\{a,b,c\}$ and let the fuzzy sets $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6$ and α_7 are defined as $\alpha_1(a)=0.3$, $\alpha_1(b)=0.4$, $\alpha_1(c)=0.5$; $\alpha_2(a)=0.6$, $\alpha_2(b)=0.9$, $\alpha_2(c)=0.5$; $\alpha_3(a)=0.2$, $\alpha_3(b)=0.2$, $\alpha_3(c)=0.2$; $\alpha_4(a)=0.4$, $\alpha_4(b)=0.4$, $\alpha_4(c)=0.5$; and $\alpha_7(a)=0.3$, $\alpha_7(b)=0.0$, $\alpha_7(c)=0.4$. Consider the double fuzzy topologies (X,τ,τ^*), (Y,η_1,η_1^*), (Y,η_2,η_2^*), (Y,η_3,η_3^*) and (Y,η_6,η_6^*) with*

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda\in\{\alpha_1,\alpha_2\}\\ 0, & \text{o.w.}\end{cases}\qquad \tau^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda\in\{\alpha_1,\alpha_2\}\\ 1, & \text{o.w.,}\end{cases}$$

$$\eta_1(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_2\\ 0, & \text{o.w.}\end{cases}\qquad \eta_1^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_2\\ 1, & \text{o.w.,}\end{cases}$$

$$\eta_2(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_3\\ 0, & \text{o.w.}\end{cases}\qquad \eta_2^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_3\\ 1, & \text{o.w.,}\end{cases}$$

$$\eta_3(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_4\\ 0, & \text{o.w.}\end{cases}\qquad \eta_3^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_4\\ 1, & \text{o.w.,}\end{cases}$$

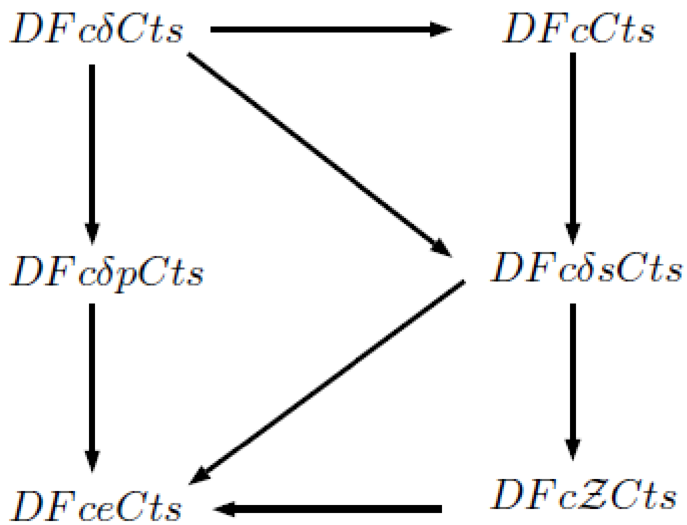
and

$$\eta_6(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_7\\ 0, & \text{o.w.}\end{cases}\qquad \eta_6^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \lambda=\alpha_7\\ 1, & \text{o.w.,}\end{cases}$$

Then the identity function (i) $f:(X,\tau,\tau^*)\rightarrow(Y,\eta_1,\eta_1^*)$ is a (i) $DFcCts$ (resp. $DFc\delta pCts$) function but not a $DFc\delta Cts$, (ii) $DFc\mathcal{Z}Cts$ but not a $DFc\delta sCts$. Since the inverse image of the fuzzy set $\underline{1}-\alpha_2$ is an $(\frac{1}{2},\frac{1}{2})$-$fo$, $(\frac{1}{2},\frac{1}{2})$-$f\delta po$ and $(\frac{1}{2},\frac{1}{2})$-$f\mathcal{Z}o$ set but not an $(\frac{1}{2},\frac{1}{2})$- $f\delta o$

and $(\frac{1}{2},\frac{1}{2})$-$f\delta so$; (iii) $f:(X,\tau,\tau^*)\to(Y,\eta_3,\eta_3^*)$ is a $DFc\delta sCts$ function but not a $DFcCts$, since the inverse image of the fuzzy set $\underline{1}-\alpha_4$ is an $(\frac{1}{2},\frac{1}{2})$-$f\delta so$ set but not an $(\frac{1}{2},\frac{1}{2})$- fo; and (iv) $f:(X,\tau,\tau^*)\to(Y,\eta_6,\eta_6^*)$ is a $DFceCts$ function but not a $DFc\mathcal{Z}Cts$, since the inverse image of the fuzzy set $\underline{1}-\alpha_7$ is an $(\frac{1}{2},\frac{1}{2})$-$feo$ set but not an $(\frac{1}{2},\frac{1}{2})$- $f\mathcal{Z}o$ set in (X,τ,τ^*).

From the above theorem and examples, the following implications are hold.



Note: $A\to B$ denotes A implies B, but not conversely.

Theorem 3.2 *Let* (X,τ,τ^*) *and* (Y,η,η^*) *be dfts's and* $f:(X,\tau,\tau^*)\to(Y,\eta,\eta^*)$ *be a mapping. Then the following statements are equivalent:*

1. f is $DFc\mathcal{Z}Cts$ function.
2. $f^{-1}(\lambda)$ is an (ι,κ)-$f\mathcal{Z}c$ set in (X,τ,τ^*) for each (ι,κ)-fo set λ in (Y,η,η^*).
3. $f^{-1}(\lambda)$ is an (ι,κ)-$f\mathcal{Z}o$ set in (X,τ,τ^*) for each (ι,κ)-fc set λ in (Y,η,η^*).
4. $f(\mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa))\leq C_{\eta,\eta^*}(f(\lambda),\iota,\kappa),\ \forall\ \lambda\in I^X$.
5. $\mathcal{Z}I_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa)\leq f^{-1}(C_{\eta,\eta^*}(\lambda,\iota,\kappa)),\forall\lambda\in I^Y$.
6. $I_{\tau,\tau^*}(\delta C_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa),\iota,\kappa)\wedge C_{\tau,\tau^*}(I_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa),\iota,\kappa)\leq f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa)),\ \forall\ \lambda\in I^Y$.
7. $\mathcal{Z}C_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa)\leq f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa))$, for each $\lambda\in I^Y$.
8. $f^{-1}(C_{\eta,\eta^*}(\mu,\iota,\kappa))\leq C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(f^{-1}(\mu),\iota,\kappa),\iota,\kappa)\vee I_{\tau,\tau^*}(C_{\tau,\tau^*}(f^{-1}(\mu),\iota,\kappa),\iota,\kappa)$ for each $\mu\in I^Y$.

Proof. (ii)⇒(iii), (v)⇒(vii), (vi)⇒(viii), (viii)⇒(ii) are direct to prove, other results are provided here.

(i)⇒(ii): Let λ be an (ι,κ)-fo set in (Y,η,η^*), f is a $DFc\mathcal{Z}Cts$ function, then we have $f^{-1}(\underline{1}-\lambda)$ is an (ι,κ)-$f\mathcal{Z}o$ set of (X,τ,τ^*). But $f^{-1}(\underline{1}-\lambda)=\underline{1}-f^{-1}(\lambda)$. Therefore $f^{-1}(\lambda)$ is an (ι,κ)-$f\mathcal{Z}c$ set of (X,τ,τ^*).

(iii)⇒(iv): Let $\lambda\in I^X$, since $\eta(I_{\eta,\eta^*}(f(\lambda),\iota,\kappa))\geq\iota$, $\eta^*(I_{\eta,\eta^*}(f(\lambda),\iota,\kappa))\leq\kappa$. Then by (iii),

$$f^{-1}(C_{\eta,\eta^*}(f(\lambda),\iota,\kappa))$$

is an (ι,κ)-$f\mathcal{Z}o$ set of (X,τ,τ^*). Since $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(C_{\eta,\eta^*}(f(\lambda),\iota,\kappa))$, we have $\mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq f^{-1}(C_{\eta,\eta^*}(f(\lambda),\iota,\kappa))$. Hence $f(\mathcal{Z}I_{\tau,\tau^*}(\lambda,\iota,\kappa)) \leq C_{\eta,\eta^*}(f(\lambda),\iota,\kappa)$.

(iv)⇒(v): For all $\lambda \in I^Y$, let $f^{-1}(\lambda)$ instead of λ in (iv), we have

$$f(\mathcal{Z}I_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa)) \leq C_{\eta,\eta^*}(f(f^{-1}(\lambda)),\iota,\kappa) \leq C_{\eta,\eta^*}(\lambda,\iota,\kappa).$$

It implies that $\mathcal{Z}I_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa) \leq f^{-1}(C_{\eta,\eta^*}(\lambda,\iota,\kappa))$.

(vii)⇒(i): Let λ be an (ι,κ)-fc set in (Y,η,η^*). Then $\underline{1}-\lambda = I_{\eta,\eta^*}(\underline{1}-\lambda,\iota,\kappa)$. By (vii), $f^{-1}(\underline{1}-\lambda) \geq \mathcal{Z}C_{\tau,\tau^*}(f^{-1}(\underline{1}-\lambda),\iota,\kappa)$. But we know that $f^{-1}(\underline{1}-\lambda) \leq \mathcal{Z}C_{\tau,\tau^*}(f^{-1}(\underline{1}-\lambda),\iota,\kappa)$. Thus, $f^{-1}(\underline{1}-\lambda) = \mathcal{Z}C_{\tau,\tau^*}(f^{-1}(\underline{1}-\lambda),\iota,\kappa)$, that is, $f^{-1}(\underline{1}-\lambda)$ is (ι,κ)-$f\mathcal{Z}c$ set. Since $f^{-1}(\underline{1}-\lambda) = \underline{1}-f^{-1}(\lambda)$, $f^{-1}(\lambda)$ is (ι,κ) - $f\mathcal{Z}o$ set. Therefore f is $DFcMc$ function.

(ii)⇒(vi): For all $\lambda \in I^Y$, since $I_{\eta,\eta^*}(\lambda,\iota,\kappa)$ is an (ι,κ)-fo set in (Y,η,η^*), by (ii), we that $f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa))$ is an (ι,κ)-$f\mathcal{Z}c$ set in (X,τ,τ^*). Hence

$$\begin{aligned} f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa)) &\geq I_{\tau,\tau^*}(\delta C_{\tau,\tau^*}(f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa)),\iota,\kappa),\iota,\kappa) \\ &\quad \wedge C_{\tau,\tau^*}(I_{\tau,\tau^*}(f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa)),\iota,\kappa),\iota,\kappa) \\ &\geq I_{\tau,\tau^*}(\delta C_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa)),\iota,\kappa) \wedge C_{\tau,\tau^*}(I_{\tau,\tau^*}(f^{-1}(\lambda),\iota,\kappa),\iota,\kappa). \end{aligned}$$

(vi)⇒(ii): For all $\lambda \in I^Y$, since $I_{\eta,\eta^*}(\lambda,\iota,\kappa)$ is an (ι,κ)-fo set in (Y,η,η^*), and let $I_{\eta,\eta^*}(\lambda,\iota,\kappa)$ instead of λ in (vi), we have that

$$\begin{aligned} &I_{\tau,\tau^*}(\delta C_{\tau,\tau^*}(f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa)),\iota,\kappa),\iota,\kappa) \wedge C_{\tau,\tau^*}(I_{\tau,\tau^*}(f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa)),\iota,\kappa),\iota,\kappa) \\ &\leq f^{-1}(I_{\eta,\eta^*}(I_{\eta,\eta^*}(\lambda,\iota,\kappa),\iota,\kappa)) \\ &= f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa)). \end{aligned}$$

Hence $f^{-1}(I_{\eta,\eta^*}(\lambda,\iota,\kappa))$ is an (ι,κ)-$f\mathcal{Z}c$ set in (X,τ,τ^*). Ξ

Lemma 3.1 *For a $DFts$ (X,τ,τ^*), every (ι,κ)-fuzzy dense set is (ι,κ)-$f\delta so$.*

Proposition 3.1 *Let $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ $DFc\mathcal{Z}Cts$ mapping and if for any fuzzy subset λ of X is (ι,κ)-fuzzy nowhere dense then f is $DFc\delta sCts$.*

Proof. Let $\tau_2(\underline{1}-\mu) \geq \iota$, $\tau_2^*(\underline{1}-\mu) \leq \kappa$. Since f is an $DFc\mathcal{Z}Cts$ mapping, then $f^{-1}(\mu)$ is an (ι,κ)-f$\mathcal{Z}$o set in (X,τ_1,τ_1^*). Put $f^{-1}(\mu) = \lambda$ is an (ι,κ)-f$\mathcal{Z}$o set in X. Hence $\lambda \leq C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa) \vee I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa)$. But $\delta I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq C_{\tau,\tau^*}(\lambda,\iota,\kappa)$, then $\delta I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa)$. Since λ is (ι,κ)-fuzzy nowhere dense and Lemma 3.1, we have $\delta I_{\tau,\tau^*}(\lambda,\iota,\kappa) = \underline{0}$. Therefore f is $DFc\delta sCts$.

Definition 3.2 *A mapping $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is called double fuzzy contra δ-open map (briefly $DFc\delta O$) if the image of every (ι,κ)-fo set of (X,τ_1,τ_1^*) is (ι,κ)-$f\delta c$ set in (Y,τ_2,τ_2^*).*

Definition 3.3 *A mapping $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is called double fuzzy contra δ-bicontinuous (briefly, $DFc\delta biCts$) if f is $DFc\delta O$ map and $DFc\delta Cts$ map.*

Theorem 3.3 *If $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ be a $DFc\delta biCts$ mapping then the inverse image of each (ι,κ)-$f\mathcal{Z}o$ set in (Y,τ_2,τ_2^*) under f is (ι,κ)-$f\mathcal{Z}c$ set in (X,τ_1,τ_1^*).*

Proof. Let f be a $DFc\delta biCts$ and $\underline{1}-\mu$ be a (ι,κ)-$f\mathcal{Z}o$ set in $(Y\tau_2,\tau_2^*)$. Then $\underline{1}-\mu \leq C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(\underline{1}-\mu,\iota,\kappa),\iota,\kappa) \vee I_{\tau_2,\tau_2^*}(C_{\tau_2,\tau_2^*}(\underline{1}-\mu,\iota,\kappa),\iota,\kappa)$.

i.e., $\mu \geq I_{\tau_2,\tau_2^*}(\delta C_{\tau_2,\tau_2^*}(\mu,\iota,\kappa),\iota,\kappa) \wedge C_{\tau_2,\tau_2^*}(I_{\tau_2,\tau_2^*}(-\mu,\iota,\kappa),\iota,\kappa)$.

$$\begin{aligned} f^{-1}(\mu) &\leq f^{-1}(C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(\mu,\iota,\kappa),\iota,\kappa)) \vee f^{-1}(I_{\tau_2,\tau_2^*}(C_{\tau_2,\tau_2^*}(\mu,\iota,\kappa),\iota,\kappa)). \\ &\leq C_{\tau_2,\tau_2^*}(f^{-1}(\delta I_{\tau_2,\tau_2^*}(\mu,\iota,\kappa)),\iota,\kappa) \vee f^{-1}(I_{\tau_2,\tau_2^*}(C_{\tau_2,\tau_2^*}(\mu,\iota,\kappa),\iota,\kappa)). \end{aligned}$$

Since f is an $DFc\delta biCts$ mapping, then f is $DFc\delta O$ map and $DFc\delta Cts$ map.

Then f is $DFc\delta sCts$ map and $DFcpCts$ map. Hence $f^{-1}(\mu) \leq C_{\tau_2,\tau_2^*}\delta I_{\tau_2,\tau_2^*}(f^{-1}(\mu),\iota,\kappa),\iota,\kappa) \vee I_{\tau_2,\tau_2^*}C_{\tau_2,\tau_2^*}(f^{-1}(\mu),\iota,\kappa),\iota,\kappa)$.

This shows that $f^{-1}(\mu)$ is (ι,κ)-f$\mathcal{Z}$o set in (X,τ_1,τ_1^*), i.e., $\underline{1} - f^{-1}(\mu) = f^{-1}(\underline{1} - \mu)$ is (ι,κ)-f$\mathcal{Z}$c set in (X,τ_1,τ_1^*)

Remark 3.2 *If $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ be a $DFc\delta biCts$ mapping. Then the inverse image of each (ι,κ)-fpo (resp. (ι,κ)-fδso) set in Y under f is (ι,κ)-f$\mathcal{Z}$c set in X.*

Theorem 3.4 *Let (X,τ_1,τ_1^*), (Y,τ_2,τ_2^*) and (Z,τ_3,τ_3^*) be dfts's. If $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ and $g:(Y,\tau_2,\tau_2^*) \to (Z,\tau_3,\tau_2^*)$ are mappings, then $g \circ f$ is $DF\mathcal{Z}Cts$ mapping if*

1. f is $DFc\mathcal{Z}Cts$ and g is $DFCts$.
2. f is $DF\mathcal{Z}Cts$ and g is $DFcCts$.
3. f is $DFc\delta biCts$ and g is $DF\mathcal{Z}Cts$ mapping.
4. f is $DF\delta biCts$ and g is $DFc\mathcal{Z}Cts$ mapping.

Proof. (i) Let $\tau_3(\mu) \geq \iota$& $\tau_3(\mu) \leq \kappa$ Since g is $DFCts$ then $\tau_3(g^{-1}(\mu)) \geq \iota$& $\tau_3(g^{-1}(\mu)) \leq \kappa$. Since f is $DFc\mathcal{Z}Cts$, then $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)$ is (ι,κ)-f$\mathcal{Z}$c set in (X,τ_1,τ_1^*). Hence $g \circ f$ is $DFc\mathcal{Z}Cts$.

(ii) Similar to (i).

(ii) Let $\tau_3(\mu) \geq \iota$& $\tau_3(\mu) \leq \kappa$. Since g is $DF\mathcal{Z}Cts$, then $g^{-1}(\mu)$ is an (ι,κ)-f$\mathcal{Z}$o set in (Y,τ_2,τ_2^*). Since f is $DF\delta biCts$, by Theorem 3.3, $(g \circ f)^{-1}(\mu)$ is (ι,κ)-f$\mathcal{Z}$o set in (X,τ_1,τ_1^*). Hence $g \circ f$ is $DF\mathcal{Z}Cts$.

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