Fluid flow and Free Convective Heat Transfer in Porous Trapezoidal Cavity with Non Uniform Heating

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Abstract: *Steady state free convection heat transfer within a porous cavity trapezoidal shape with sinusoidal heating at constant aspect ratio. The left sidewall of the trapezoidal enclosure is heated and right sidewall of the trapezoidal shape is cooled. The other two parallel top surface and bottom surface of the cavity are adiabatic. The thermal parameters (Stream functions, Isotherms and Nusselt numbers) are performed for range of the Rayleigh number* 10^2 *to 1500 for an aspect ratio (H/L) of 0.5 of cavity .The present study is carried out for the trapezoidal cavity being heated sinusoidal with sidewall. It is found that constant or uniform temperature at right side wall of the cavity obtained higher Nu as compared to non uniform temperature cases, resulting in better enhancement of heat transfer rate and the results showed excellent agreement with other publications.*

Keywords: Trapezoidal cavity, Natural convection, Porous medium, Non-uniform heating and Nusselt number.

I. INTRODUCTION

Fluid flow and heat transfer analysis in uniformly and non uniformly heated porous media has become a separate topic for research in last three decades in view of its importance in various applications such as power plant waste in nuclear power plants, fiber insulations solar collectors etc.

Various authors studied wide range of the applications of this problem. The extensive of research in porous enclosure has been investigated recently by Hussian [1] and Nield and Darcy [2] on flow through porous beds identified permeability as a property of porous media. Baytas and Pop [3], Basak et al. [4], Sathiyamoorthy et al. [5], carried out studies on free convection inside the different cavities having different heat flux and temperature boundary conditions.

Free convection inside the different structure of enclosures, 3 noded element non-rectangle, or different cavities like wavy or sinusoidal surface, with different boundary conditions are studied to analyze the heat transfer and fluid flow. Peric [6] studied free convective heat transfer in non square cavity using a finite control volume method and present prediction agree well with the result of Peric [6].

Irfan [7] carried out heat transfer in triangular and rectangular porous cavity subjected to different heat transfer boundary conditions with finite element governing equations. Basak et al. [8] carried out investigation on heat, fluid flow of natural convection flows with in trapezoidal cavities. Sehrina et al, [9] investigated about the free convective buoyant transfer in trapezoidal cavity with wavy or sinusoidal top wall. The top surface is cold while the bottom surface is sinusoidal heated and sidewalls are maintained adiabatic.

Aparna and Seetharamu [10] studied simulations using finite element method with triangular elements to investigate the convective heat transfer of constant and non-uniform heating from bottom wall within a trapezoidal cavity of constant inclination angle. Numerical study has been carried out different number of Rayleigh number. Quantitative results are obtained in terms of isotherm contours, Nusselt numbers and stream functions. For hot and cold surfaces with increasing *Ra* average nusslet number increases.

Kaminski et al. [11] analyzed numerically the steady, laminar, free convection flow in a regular cavity. One vertical wall of the enclosure is more thickness, with a finite thermal conductivity, while remaining walls are taken to be of zero thickness. The problem is conjugate or mixed heat transfer to determining the heat transfer effect in the wall on the buoyant flow in square cavity.

Bhuvaneswari et al. [12] have considered rectangular domain with different side ratio with thermal walls. Numerical and computational study has been carried with heat transfer rate approaches to decreasing the aspect ratio.

Hammami [13] presented quantitative results on three-dimensional study of combined heat and concentration difference or mass transfer by free convection occurring in a trapezoidal cavity. They solved the governing equations by using a Matlab.

Wiratchada et al. [14] investigated free convection in porous cavities with desecrate heating positions on bottom and sidewalls. It is found that, for higher Rayleigh numbers convection effect is more on sidewall heating conditions.

Varol et al. [15] numerically performed fluid flow and heat transfer in a porous cavity with constant temperature and non-isothermally bottom wall heated a triangular cavity is analyzed using finite difference method. Darcy governing law was used to generate the codes of porous medium filled inside a cavity. Dimensionless heat transfer function was used to analyze the heat transport due buoyancy effects.

Pakdee et al [16] studied transient boundary conditions on buoyant heat transfer through porous rectangular enclosure due to top surface discrete convective heat transfer. The results are obtained in terms of non-dimensional numbers as a result of higher Rayleigh indicates convection effect is more.

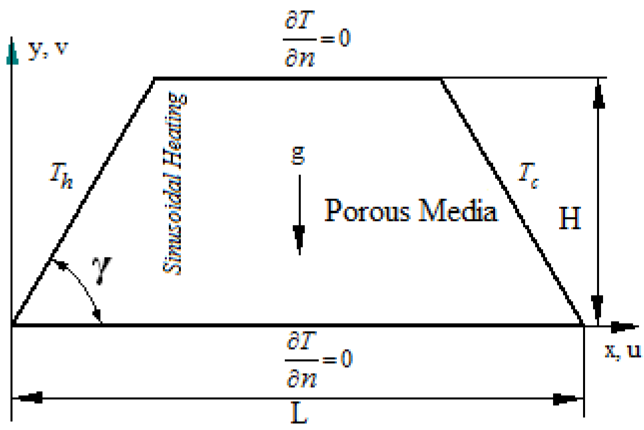

Fig. 1: *Physical domain*

II. GOVERNING EQUATIONS

The model considered in the present paper consists of an porous trapezoidal cavity as shown in Fig. 1 along with the boundary conditions and coordinates. The cross section view of the working model is a trapezoid of height *(H)*, Length of cavity *(L)*, with a left inclined hot wall with uniformly varying boundary conditions, right side wall is cold and the remaining two parallel plane walls of enclosure are adiabatic.

For above working model following assumptions are made:

- The working medium like porous medium fallows the governing law of Permeability (Darcy law).
- The established fluid flow is laminar.
- The working medium (Porous) is saturated with fluid.
- The local thermal equilibrium obtained between fluid and medium with in domain.
- Working fluid is Single Phase and thermal properties are same in all directions.(isotropic)
- Working medium properties are remains constant except the free convection parameter as density.

The basic equations for present study as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Darcy equation

$$u=\frac{-K}{\mu}\frac{\partial p}{\partial x} \tag{2}$$

The velocity in X- direction can be described by

$$u=\frac{-K}{\mu}\frac{\partial p}{\partial x} \tag{3}$$

Velocity in vertical direction is given by

$$v=\frac{-K}{\mu}\left(\frac{\partial p}{\partial y}+\rho g\right) \tag{4}$$

The permeability K of porous medium is given by

$$K=\frac{D_P^2\phi^3}{180(1-\phi)^2} \tag{5}$$

The density variation within the porous cavity is given by

$$\rho=\rho_\infty[1-\beta_T(T-T_\infty)] \tag{6}$$

Momentum equation:

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\frac{Kg\beta_T}{\gamma}\frac{\partial T}{\partial x} \tag{7}$$

Energy equation;

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{8}$$

The continuity equation follows following stream functions such as

$$u=\frac{\partial \psi}{\partial y} \tag{9}$$

$$v=-\frac{\partial \psi}{\partial x} \tag{10}$$

Non-dimensional temperature $\theta=\left(\frac{T-T_C}{T_h-T_C}\right)$

Non-dimensional Stream function $\psi=\frac{\psi}{\alpha}$

Non-dimensional Height $Y=\frac{y}{L}$

Non-dimensional Width $X=\frac{x}{L}$

Modified Rayleigh number $Ra=\frac{g\beta_T\nabla TKL}{\nu\alpha}$ (11)

Non-dimensional equations for porous cavities are

$$\left(\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}\right)=-Ra\frac{\partial\theta}{\partial X} \tag{12}$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2} \tag{13}$$

III. BOUNDARY CONDITIONS

The modified *Ra* gives value of dimensionless stream function at no slip boundary conditions as follows

$$\left.\begin{aligned} &\text{Left side hot wall } \theta(X,0) = \sin(\pi X) \\ &\text{Right side cold wall } \theta_c = 0. \\ &\text{Top and Bottom parallel walls } \frac{\partial \theta}{\partial n} = 0 \end{aligned}\right\} \quad (14)$$

IV. NUMERICAL SOLUTIONS

4.1 Stream function and temperature distributions

Galerikin's Residual finite element method with no slip boundary conditions are used to solve Darcy and energy equations numerically. Computational results for stream functions and isotherms for buoyant heat transfer were obtained for modified *Ra* numbers .To investigate the buoyant heat transfer parameters in porous medium to generate a contours by using MATLAB Code.

4.2 Nusselt number

The fluid flow and convective heat transfer performance in terms of the local Nusselt number and average Nusselt number can be written as;

$$Nu = -\frac{\partial \theta}{\partial n} \quad (15)$$

Average *Nu* is obtained by integrating the local *Nu* and is given by On Hot wall**,**

$$Nu_{avg} = \int_0^1 NudX = U = V = (0, L) = T_h \quad (16)$$

On Cold wall,

$$Nu_{avg} = \frac{1}{0.3}\int_{0.35}^{0.65} NudX = U = V = T_C \quad (17)$$

V. VALIDATION OF NUMERICAL RESULT

5.1 Verification of present Methodology

Present work has been validated against Aparna and Seetharamu [10] buoyant heat transfer flow in a Trapezoidal Enclosure with bottom wall with uniformly heating, uniformly cooled top wall and non parallel sidewalls are adiabatic has been performed numerically. The computations have been investigated for different *Ra* vary from *250* to *1500* results are obtained in the form of isotherms, stream functions and variations of overall Nusselt number.

5.2 Grid Test

The grid convergence test has been investigated with various mesh size of a porous trapezoidal cavity [10] for regular mesh with triangular element have been investigated using Finite Element Method.

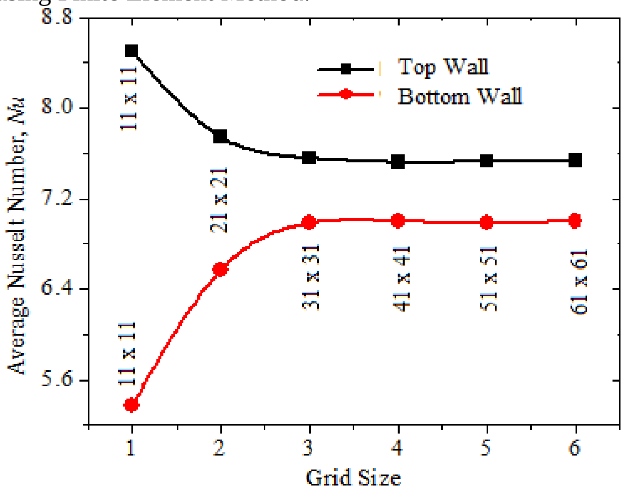

Fig. 2: *Grid independence study*

The grid convergence was studied using six different sizes of11 x 11, 21 x 21, 31 x 31,41 x 41, 51 x51 and 61 x 61.Fig. 2 shows grid independence study of the average Nusselt numbers with *Ra =1000* for left side wall uniform heating. It is observed that Average Nu for 11 x 11 is about 4.68 and is rising as the grid independence is achieved with a 41 x 41 grid. Due to above statements, 41×41 grids is used for all further computations.

6.3 Heat transfer rates: Local Nusselt numbers

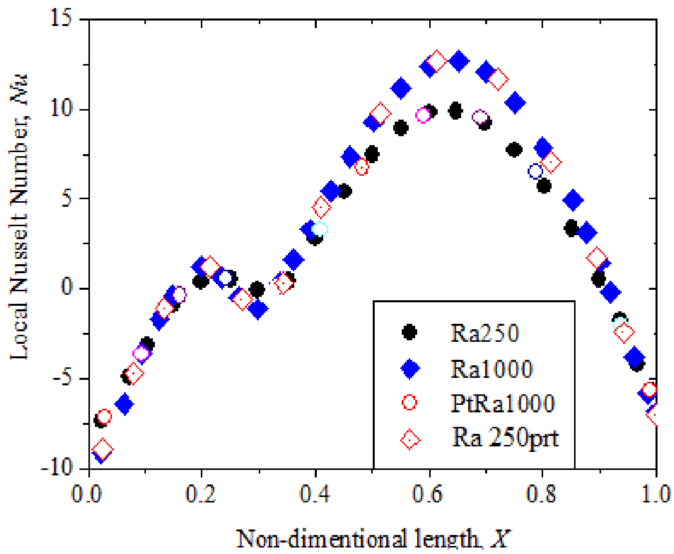

Fig 3: *Distribution of Local Nu on heated surface subjected to non uniform temperature variation for Ra =250 to 1000.*

Fig.3 displays the effect of *Ra* (250 and 1000) on local *Nu* at the heated wall. The display shows non uniform temperature distribution, but asymmetrical nature is obtained for heated surface. Two consecutive apexes obtained one is at X=0.21 and the other at X=0.7 Local Nusselt number begins with negative magnitude and to achieve positive curvature.

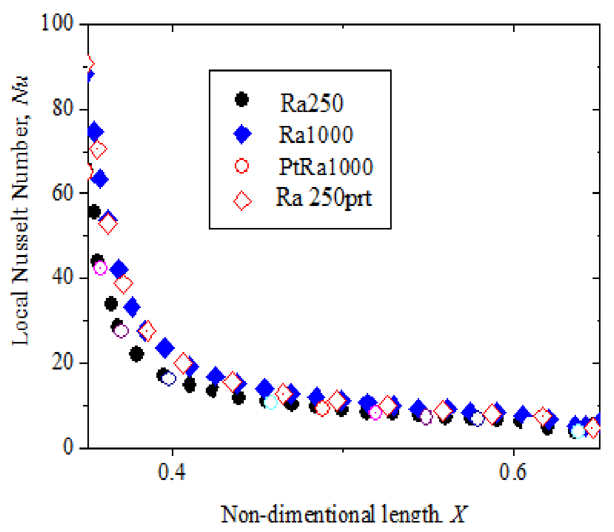

Fig 4: *Distribution of Local Nu on cold surface subjected to non uniform temperature variation for Ra =250 to 1000.*

Fig.4 displays the effect of *Ra* on local *Nu* at the top wall. The local *Nu* consistently decrease along top wall for *Ra =250* to *1000*.The local Nusselt numbers values increases with increase in *Ra*. For lower *Ra*, Local Nusselt number magnitudes are less because of conduction dominated heat transfer.

Table 1 *Validation of Average Nu with sinusoidal Bottom wall heating is carried out with Published data of Aparna and Seetharamu [10].*

Ra	$\overline{Nu}$ Present Study	$\overline{Nu}$ Published Data [10]	**Discrepancy in %**
250	3.551	3.544	0.309
500	4.581	4.601	0.436
1000	5.843	5.831	0.205

The computations of Average *Nu* for Rayleigh numbers, *Ra* (250-1000) with present code has been computed with sinusoidal bottom heating .the obtained values have been compared with that of Aparna and Seetharamu [10].It is seen that there is good agreement with maximum error incurred between two are less than 1%.

Table 2 *Validation of Average Nu with sinusoidal Top wall heating is carried out with Published data of Aparna and Seetharamu [10].*

Ra	$\overline{Nu}$ Present Study	$\overline{Nu}$ Published Data [10]	**Discrepancy in %**
250	2.450	2.442	0.3265
500	3.551	3.568	0.4787
1000	4.742	4.749	0.1476

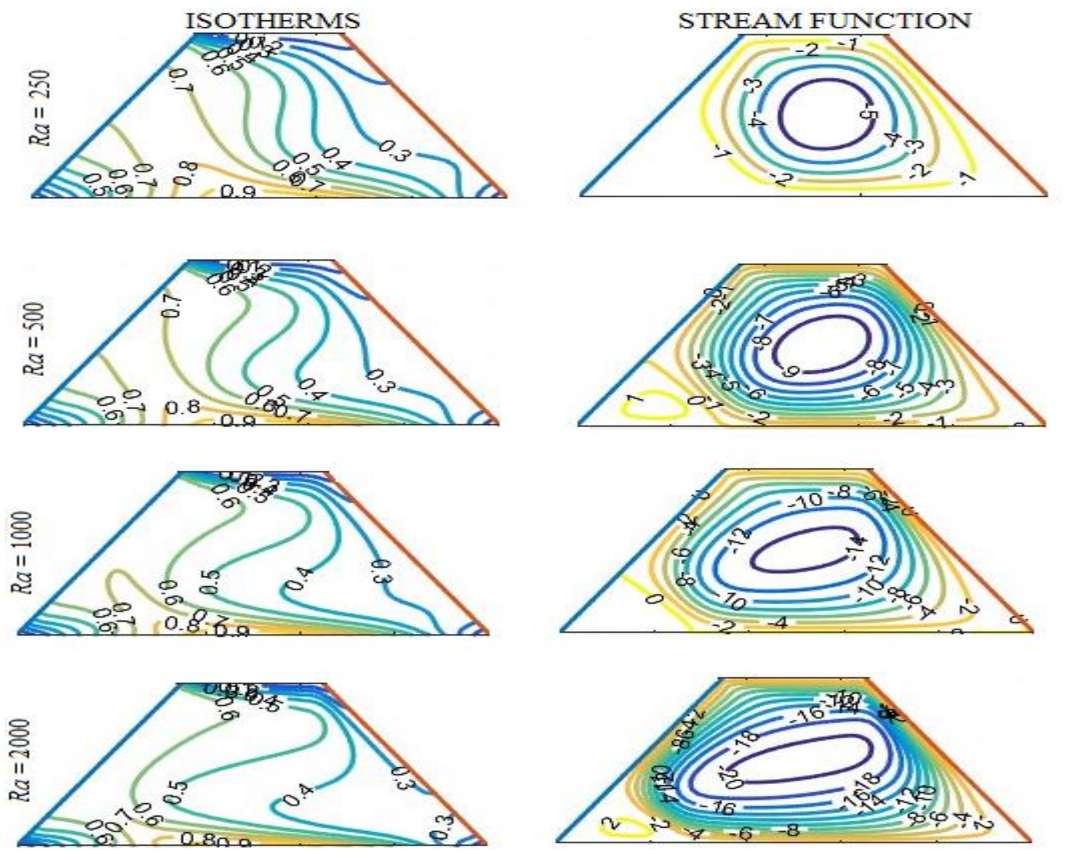

Fig 5*: Bottom surface subjected to non uniform temperature.*

Figs shows isotherms and stream functions for *Ra = 250,500,1000 and 2000* the heated surface subjected to non uniform temperature. It is observed that at very low *Ra* equal to 250, indicating buoyant heat transfer dominance and in a stream function small single loop formed at *Ra = 500* at the heated wall of the enclosure . Stream function magnitudes are negative values with inner circular loop when *Ra = 250,500* and when *Ra = 2000,* elliptical loop formed and elliptical shape spread over a cavity.

Isotherms profiles are non symmetric about vertical axis with positive values and single lines starting from heated wall moves towards right inclined wall with an high temperature as it travels form left inclined wall to right inclined wall.

When *Ra =250*, It is found that isotherms are less curvature compare to *Ra =2000* hence at high Rayleigh number signifying convection dominance.

VI. RESULTS AND DISCUSSIONS

The trapezoidal cavity with $\gamma = 55^0$ (Fig.1) has been considered for numerical studies. The computational domain consists of triangular elements which correspond to 41 x 41 grids. Numerical salutations are obtained for various values of Ra = 250,500 and 750 with non uniform heating left side wall where as two horizontal top and bottom walls are well insulated and right side wall is cooled. In order to examine the accuracy of numerical procedure, we have benchmarked our code based on the mesh size for the porous trapezoidal enclosure [10].Also, the result is good agreement with an previous numerical procedure [10] for porous trapezoidal cavity with hot wall.

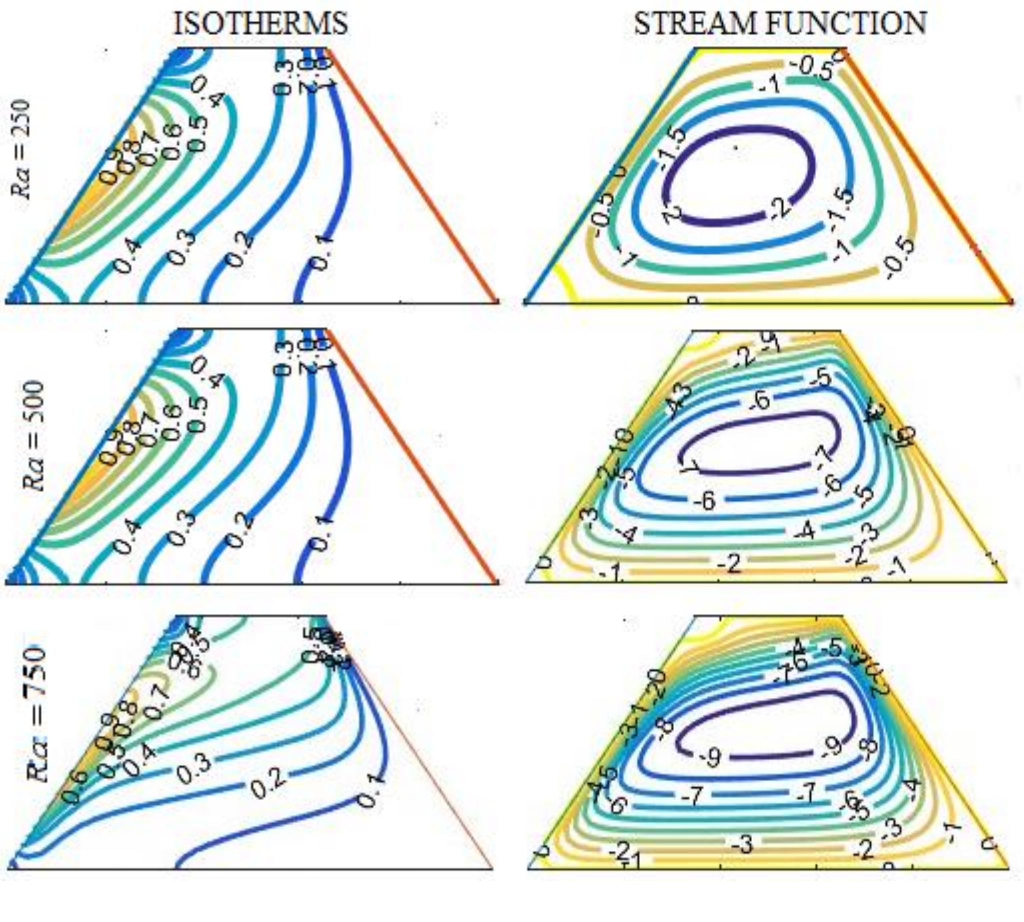

Fig. 6: *Streamlines and Isotherms for left side wall maintained at non uniform temperature condition for Ra =250, Ra=500, Ra=750.*

Fig. 6 displays the impact of Rayleigh number on isotherms and stream functions for $\gamma = 55^0$. It is examine that *Ra* reflects the impact of the buoyant force in free convection problems. Due to low *Ra* numbers, the density of the isotherms at heated wall and cold wall boundaries decreases. The presence of significant convection is also exhibited in isotherm contour lines which start getting deformed and pushed towards the upper surface.

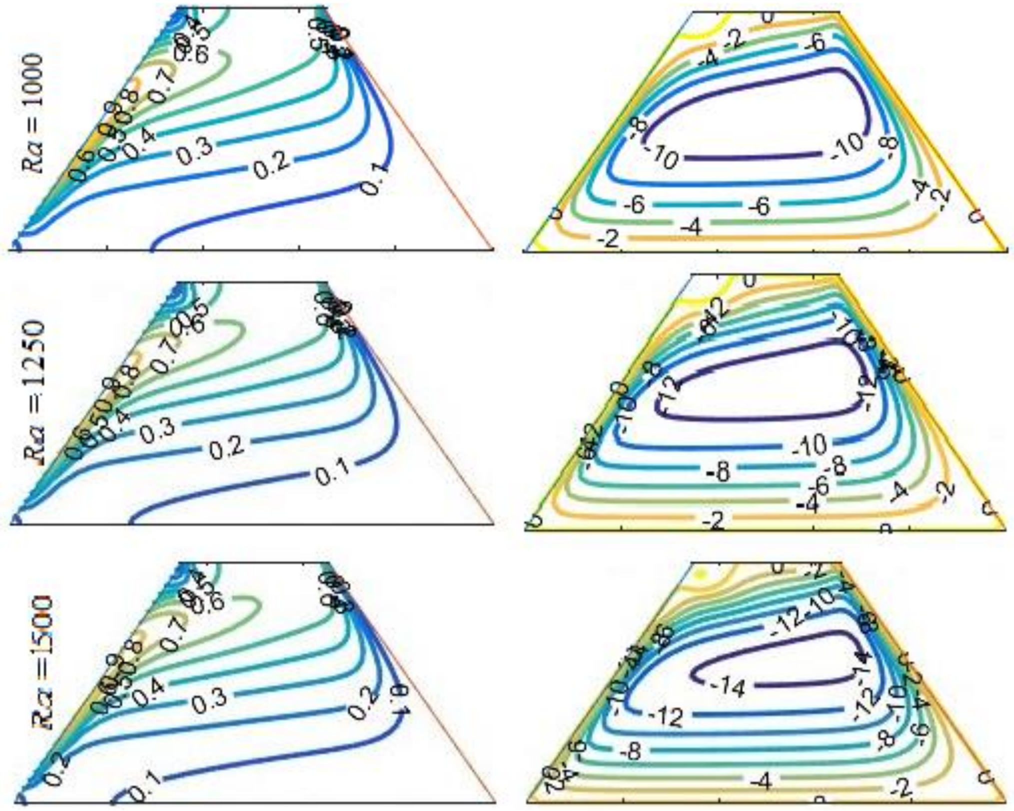

Fig. 7: *Streamlines and Isotherms for left side wall maintained at non uniform temperature condition for Ra =1000, Ra=1250, Ra=1500.*

Fig. 7 show the impact of Rayleigh number on isotherms and stream functions for $\gamma = 55^0$. It is examine that *Ra* reflects the impact of the buoyant force in free convection problems. Due to high *Ra* numbers, the density of the temperature contours at heated wall and cold wall boundaries enhanced. The intensity of circulation is greater near the hot wall and least near the walls due to no slip conditions. Stream function will be order of negative magnitude with inner loop with low Ra and lower magnitude of stream functions. Elliptical nature obtained for higher Ra with maximum magnitude of stream functions and elliptical loop spreads over the enclosure for *Ra = 1500*.

VII.CONCLUSION

In the current study, a numerical simulation of buoyant heat transfer inside the porous trapezoidal cavity, in the presence of the convective heat transfer effect was presented. The investigations have been carried using triangular elements for the left sidewall subjected to Sinusoidal heating boundary conditions and cold right side wall. The upper and lower surfaces are insulated. For this study, it can be concluded that:

- A clear reduction in both local *Nu* and average *Nu* number is obtained by increase the Rayleigh numbers.
- A good enhancement of average *Nu* is obtained by increase the Rayleigh number.
- The conduction dominated energy transfer occurred up to low Rayleigh numbers.
- Compare to uniform heating, the stream function value increases for Sinusoidal heating boundary conditions.
- The average *Nu* increase consistently with increase of Ra for right side heating wall and is more for uniform temperature than sinusoidal varying temperature case for the walls.

VIII. REFERENCES

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