Design of One Plan Suspension System with Multiple Repetitive Groups Sampling Plan through Minimum Sum of Risk.

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Abstract

Acceptance sampling systems are advocated when small sample size are necessary or desirable towards costlier testing for product quality. When single plan is used with suspension rule, the system is called **One-Plan (OP) Suspension System**. In OP suspension system, a lot-by-lot sampling plan is used to decide whether individual lots are accepted or rejected. This paper provides a new procedure for Designing and Selection of One Plan Suspension System with Multiple Repetitive Group Sampling Plan as reference plan. Tables and procedures are also provided for the selection of parameter for the plan. Numerical illustrations are also provided for the shop- floor applications of these procedures.

Keywords:

Suspension System, Sampling Plan, Gamma-Poisson distributions, stopping rule, Minimum Sum of Risk.

1. Introduction

Acceptance sampling is a tool for consumer to reject bad lots as well as producer to quicken the process control. In a progressive atmosphere of production with increased chances of occurrence of non-conforming material, statistical process control optimizes the process capability and acceptance sampling acts logically to prevent passing out non-conforming units.

This paper is concerned with acceptance sampling plans when small samples are necessary or desirable, for example, when production quantities are small or when inspection is either costly or destructive. Under these conditions, an attribute plan with small sample size is not very effective, since the discrimination between good and bad quality is not sufficient. Nor does lot-by-lot inspection provide an incentive for the producer to turn out consistently good quality.

Suresh and Indira (2013) have studied the Construction and Selection of One Plan Suspension System with Bayesian Chain Sampling Plan through Acceptable and Limiting quality levels. She has also studied Designing of One Plan Suspension System with Repetitive Deferred sampling Plan indexed through Probability of acceptance.

2. Selection of Sampling Plan

The concept of Repetitive Group sampling (RGS) Plan introduced by Sherman (1965) in which acceptance or rejection of a lot is based on the repeated sample results of the same lot. Later, Gauri Shankar and Joseph (1993) have proposed a new repetitive Group sampling plan as an extension of the conditional repetitive group sampling plan in which acceptance or rejection of a lot on the basis of repeated sample results is dependent on the outcome of inspection under Repetitive Group Sampling inspection system of the preceding lots. Further they derived the formulae for OC and ASN functions. An attempt has been made to model and analyze the dynamics of the proposed inspection system through GERT approach.

Suresh and Saminathan (2007) have given a procedure to define Multiple Repetitive Group sampling plan indexed with MAPD and MAAOQ. Suresh and Kaviyarasu (2011) have studied QSS-1 with Multiple RGS as reference plan indexed with Acceptable Quality level (AQL). Limiting Quality Level (LQL) , Indifference quality Level (IQL) and its operating Ratio. Poisson unity values have been tabulated to facilitate the operation and construction of the plan. Illustrations are also provided for selection of plan parameters.

2.1 Operating Procedure

1. Draw a random sample of size n and determine the number of defectives (d) found there in.
2. Accept the lot if $d \leq c_1$ (or)
   Reject the lot if $d > c_2$
3. If $c_1 < d < c_2$, accept the lot provided "i" proceeding or succeeding lots are accepted under RGS inspection plan, otherwise reject the lot

Thus MRGS plan is characterized through four parameters, namely, n, c_1, c_2 and the acceptance criterion i. Here it may be noted that when $c_1 = c_2$, the resulting plan is simple single sampling. Also for i=0, we have the RGS plan of Sherman (1965). It may further be noted that the conditions for application of the proposed plan is same as Sherman's RGS plan.

The operating characteristic function Pa (p) of Multiple Repetitive Group Sampling plan is derived by Shankar and Joseph (1993) under Poisson model as

$$P_a(p) = \frac{P_a(1-P_c)^i}{(1-P_c)^i - P_c P_a^i}$$

Where $$P_a = P[d \leq c_1] = \sum_{x=0}^{c_1} \frac{e^{-x} x^r}{x!}$$

$$P_r = 1 - \sum_{x=0}^{c_2} \frac{e^{-np}(np)^x}{x!}$$

$$P_c = P[c_1 < d < c_2] = \sum_{x=0}^{c_2} \frac{e^{-x} x^r}{x!} - \sum_{x=0}^{c_1} \frac{e^{-x} x^r}{x!}$$ and x=np.

3. One Plan Suspension system

The suspension principle to acceptance sampling system. A suspension rule is a procedure used to decide when to suspend inspection of a production process, where product is submitted for inspection in lots. The decision to suspend is based on the observed sequence of lot acceptance and rejections.

A suspension rule, which is designated (j,k), $2 \leq j \leq k$ is a rule of suspending inspection based on finding j lot rejections in k or less lots. Given j and k at least j lots must be inspected before a decision is possible upon the beginning of a new process or from the time of the last suspension. A suspension system is a combination of suspension rule and a single lot-by-lot sampling plan or pair plans. When a single plan is used with a suspension rule it is called One Plan (OP) Suspension System and when two plans, Tightened and Normal are used, it is called Two Plan (TP) Suspension System.

3.1 Average Run Length

According to Troxell (1972) the expected time for suspension or average run length of a rule is important in the evaluation of the suspension system. The average run length of the suspension rule (j,k) designated as ARL (j,k) can be calculated in the following way.

First, the expected number of lot rejections until suspension is calculated. Since lot rejections are interspaced with lot acceptances, the second step is to find the total expected number of lots inspected, including the rejected lot, between successive lot rejections, the ARL equals the sum of the total number of lots inspected until suspension. It is shown that, in fact the total number of inspected lots between consecutive rejections are independently and identically distributed for all rejections so that:

ARL (j,k) = (Total number of inspected lots between two Rejections) x

(Expected number of rejection until suspension.) (3.1.1)

Using this fact, for j=2, the expression is given by a single term and for j=3, the result is best expressed in the form of a continued fraction, which is found by solving for the stationary distribution of a particular Markov chain. For higher rules, a discussion is given indicating the method of solving for the expected number of rejections until suspension.

3.2 Operating Characteristic Curve

A different type of OC curve which has features not common to type B OC curve (1959) has been used here to study the suspension system. Since ARL for the rule (j,k) is some function of incoming quality p, this correspondence allows an operating characteristic to be plotted in the following way . In a large number of lots N, the number of lots for which the process is judged conforming, that is the number of lots for which suspension does not occur, is given approximately by N(1-1/ARL). Therefore (1-1/ARL) is interpreted as the average fraction of lots for which the process is acceptable, or the probability of accepting the process. This value is denoted as P_A,

$$P_A(j,k) = 1 - 1/ARL(j,k)$$

and hence $$P_A(2,k) = \frac{1 + P_a - P_a^{\,k}}{2 - P_a^{\,k-1}}$$

$$P_A(2,\infty) = \frac{1 + P_a}{2}$$

The OC Curve is a graph of P_A as a function of fraction defective.

4. Selection of Minimum Risk OPSS with Multiple Repetitive Group Sampling Plan

Table 1 is used to select a OPSS with Multiple Repetitive Group Sampling plan as reference for given AQL (p_1) and LQL (p_2) which involves minimum sum of risks. For the plan of Table 2.8.1, producer and consumer risk will be at most 10% each against fixed values of the operating ratio p_2/p_1.

Table 1 gives the parameters c_1, c_2, k and producer and consumer risks (α and β respectively) in the body of the table against the product of sample size (n) and AQL (p_1). With the given p_1, p_2, α and β, one can find OPSS with MRGS follows.

The following procedure is used for selecting plans for given AQL, LQL, α and β.

1. Compute the operating ratio p_2/p_1.
2. With the computed value of p_2/p_1, enter table in the row headed by p_2/p_1, which is equal to or just smaller than the computed ratios.
3. For determining the parameters c_1, c_2, and k of the One plan Suspension System with Multiple Repetitive Group Sampling Plan, one proceeds from left to right in the row identified in step2 such that the tabulated producer and consumer risks are equal it or just less than the desired value.
4. The sample size n is obtained as n= p_2/p_1, where $n\mu_1$ values are given in the column heading corresponding to the parameters 'c_1, c_2 and k' identified in step 3.

Example 1

If one fixes p_1=0.01, p_2 =0.34, α=0.05 and β=0.10, from Table 1, one can find the system designated as the One Plan Suspension System with Multiple Repetitive Group Sampling Plan involving minimum sum of risks as follows

1. p_2/p_1 =34
2. Tabulated p_2/p_1= 34
3. Corresponding to k=2, c_1=2 and c_2=15, give in the body of the table, one obtains α=0.25, β=0.27 against the desired value α =0.05, β=0.10.
4. n = np_1/p_1= 0.05/0.01=5.

Construction of Tables

The expression for the OC function of One Plan Suspension System with Multiple Repetitive Group Sampling Plan as reference plan is defined as

$$P_A(2.k)=\frac{1+P_a(1-P_c)^i+P_cP_a{}^i/(1-P_c)^i-\left(P_a(1-P_c)^i+P_cP_a{}^i/(1-P_c)^i\right)^k}{2-\left(P_a(1-P_c)^i+P_cP_a{}^i/(1-P_c)^i\right)^{k-1}} \quad (4.1)$$

$$P_a(p)=\frac{P_a(1-P_c)^i+P_cP_a{}^i}{(1-P_c)^i} \quad (4.2)$$

Where
$$P_a=P[d\le c_1]=\sum_{r=0}^{c_1}\frac{e^{-x}x^r}{r!} \quad (4.3)$$

$$P_c=P[c_1<d<c_2]=\sum_{r=0}^{c_2}\frac{e^{-x}x!}{r!}-\sum_{r=0}^{c_1}\frac{e^{-x}x^r}{r!} \quad (4.4)$$

and $x = np$

The expression for the sum of the producer and consumer risk is given as

$$\alpha + \beta = [1 - P_a(p_1)] + P_a(p_2) \tag{4.5}$$

If the operating ratio p_2/p_1 and np1 are known, then the expression for np_2 can be written as

$$np_2 = \left({p_2}/{p_1} \right)(np_1) \tag{4.6}$$

The parameter c_1, c_2, and k corresponding to the minimum $[1 - P_a(p_1)] + P_a(p_2)$ are obtained through searching for k=2(0.25), c_1=2(0.27) and c_2=15(0.27) with the help of solving expression through computer program.

The values in Table 1 gives producers and consumer risks which are obtained corresponding to the values of c_1, c_2, and k for which the sum of risk is minimum.

Conclusion

Acceptance sampling is the technique, which deals with the procedures in which decision to accept or reject lots or process based on their examination of past history or knowledge of samples. The present work mainly relates to the Designing of performance One Plan Suspension System with Multiple Repetitive Group Sampling Plan using Minimum Sum of Risk. These types of plans are very much essential for the engineers to accept or reject the lots having at least one defective in the lot. The emphasis in the present work is that the selection of a sampling system with this procedure is more advantages for both producer and consumer with less inspection cost.

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TABLE 1 ONE PLAN SUSPENSIONS SYSTEM WITH MULTIPLE REPETITIVE GROUPS SAMPLING PLAN THROUGH MINIMUM SUM OF RISK

np2 / OR	**0.05**	**0.15**	**0.25**	**0.35**	**0.45**	**0.55**	**0.65**
44	2,0,0 0.25,0.33	3,0,1 0.25,0.25	4,0,2 0.15,0.35	5,0,3 0.18,0.32	6,0,4 0.20,0.30	7,0,5 0.35,0.15	8,0,6 0.34,0.16
43	9,1,1 0.33,0.35	10,1,2 0.25,0.26	11,1,3 0.35,0.15	12,1,4 0.25,0.25	13,1,5 0.32,0.18	14,1,6 0.34,0.16	15,2,2 0.30,0.20
42	16,2,3 0.44,0.44	17,2,4 0.25,0.28	18,2,5 0.25,0.25	19,2,6 0.15,0.35	20,3,3 0.16,0.34	21,3,4 0.22,0.28	22,3,5 0.32,0.18
40	23,3,6 0.50,0.50	24,3,7 0.34,0.25	25,4,4 0.24,0.27	26,4,5 0.23,0.27	27,4,6 0.35,0.15	28,4,7 0.30,0.20	29,4,8 0.28,0.22
41	30,5,5 0.50,0.49	31,5,6 0.35,0.39	32,5,7 0.21,0.23	33,5,8 0.25,0.25	34,6,6 0.15,0.35	35,6,8 0.20,0.30	36,6,10 0.27,0.23
39	2,0,0 0.45,0.55	3,0,1 0.60,0.40	4,0,2 0.38,0.62	5,0,3 0.57,0.43	6,0,4 0.30,0.70	7,0,5 0.35,0.65	8,0,6 0.56,0.44
38	9,1,1 0.58,0.42	10,1,2 0.65,0.35	11,1,3 0.62,0.38	12,1,4 0.35,0.65	13,1,5 0.57,0.43	14,1,6 0.55,0.45	15,2,2 0.38,0.62
37	16,2,3 0.50,0.50	17,2,4 0.57,0.43	18,2,5 0.52,0.48	19,2,6 0.62,0.38	20,3,3 0.56,0.44	21,3,4 0.70,0.30	22,3,5 0.45,0.55
36	23,3,6 0.42,0.58	24,3,7 0.55,0.45	25,4,4 0.70,0.30	26,4,5 0.52,0.48	27,4,6 0.55,0.45	28,4,7 0.58,0.42	29,4,8 0.57,0.43
35	30,5,5 0.43,0.57	31,5,6 0.58,0.42	32,5,7 0.35,0.65	33,5,8 0.56,0.44	34,6,6 0.62,0.38	35,6,8 0.57,0.43	36,6,10 0.60,0.40
34	2,0,0 0.56,0.44	3,0,1 0.60,0.40	4,0,2 0.48,0.52	5,0,3 0.57,0.43	6,0,4 0.38,0.62	7,0,5 0.35,0.65	8,0,6 0.58,0.42
33	9,1,1 0.38,0.62	10,1,2 0.57,0.43	11,1,3 0.43,0.57	12,1,4 0.56,0.44	13,1,5 0.60,0.40	14,1,6 0.40,0.60	15,2,2 0.62,0.38
32	16,2,3 0.55,0.45	17,2,4 0.35,0.65	18,2,5 0.60,0.40	19,2,6 0.38,0.62	20,3,3 0.55,0.45	21,3,4 0.57,0.43	22,3,5 0.38,0.62
31	23,3,6 0.48,0.52	24,3,7 0.45,0.55	25,4,4 0.57,0.43	26,4,5 0.48,0.52	27,4,6 0.35,0.65	28,4,7 0.58,0.42	29,4,8 0.38,0.62
30	30,5,5 0.65,0.35	31,5,6 0.35,0.65	32,5,7 0.44,0.56	33,5,8 0.58,0.42	34,6,6 0.62,0.38	35,6,8 0.56,0.44	36,6,10 0.60,0.40
29	2,0,0 0.50,0.50	3,0,1 0.35,0.65	4,0,2 0.57,0.43	5,0,3 0.35,0.65	6,0,4 0.56,0.44	7,0,5 0.38,0.62	8,0,6 0.43,0.57
28	9,1,1 0.44,0.56	10,1,2 0.58,0.42	11,1,3 0.38,0.62	12,1,4 0.40,0.60	13,1,5 0.35,0.65	14,1,6 0.60,0.40	15,2,2 0.57,0.43