CONSTRUCTION AND SELECTION OF CHAIN SAMPLING PLAN

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ABTRACT

This paper introduces a new method for selection of chain sampling plans (0,1) indexed through quality decision regions. In this chapter, we discusses two sets of criteria for constructing ChSP-(0,1) plans. The first case is where plans are constructed given the quality decision region. In the second case, plans are constructed given the probability quality regions. Tables are provided for the easy selection of the sampling plan.

Key Words: Acceptance Sampling plan, ChSP-(0, 1), Quality Region, Inflexion point, ChSP-1, MAPD.

INTRODUCTION

Statistical method for quality improvement is the statistical process control, design of experiments and acceptance sampling. In addition to these techniques, a number of other statistical tools are useful in analyzing quality problems and improving the performance of production processes. Quality Decision Region (QDR) indexed plan provides a method for designing sampling plan based on range of quality, instead of point wise description about quality. The QDR indexed plan has been considered for Construction and Selection of Chain Sampling Plan (0, 1) which provides higher probability of acceptance compared with plan designed through quality levels. QDR proposes wider potential applicability in industry ensuring higher standard of quality attainment for product. This paper provides a new procedure for Selection of Chain Sampling Plan (0, 1) indexed through Quality Decision Regions. New operating ratios are introduced towards designing such sampling plan. A method for selection ofChSP (0, 1) plan based on range of quality instead of point wise description of quality by using the method of quality intervals sampling is also provided.

REVIEW OF LITERATURE

The concept of chain sampling inspection plans was proposed by Dodge [6]. The ChSP-1 Plans are applicable for both smaller and larger samples. Further Frishman [7] has proposed two approaches to the use of other acceptance numbers different from 0 and 1 of ChSP-1 plans which employ modified procedural rule including conditional cumulation and rejection on basis of the results of single sample.

Dodge and Stephens [5] proposed the generalized family of two – stage chain sampling plan which is the extension of ChSP-1 plan. They presented the expression for OC curves which was characterized by acceptance numbers C_1 =0 and C_2 =2. The chain sampling plan – (0,1) developed by Dodge and Stephens [6] is a generalization of Dodge's [5] ChSP -1. ChSP - (0, 1) can be useful when sample must necessarily be small (e.g. when tests are costly). The greater generality in the choice of parameters in the ChSP - (0, 1) allows for flexibility in matching these plans to other plans, and allows improved discrimination between good and bad quality.

Operating Procedure

A Chsp-(0, 1) plan is implemented in the following way:

1. A random sample of size n units is taken from each successive lot. The number of non conformities in each sample is recorded, as well as the cumulative number of non conformities found so far.
2. Accept the lot associated with each new sample as long as non conformity is found.
3. Once k_1 lots have been accepted, accept subsequent lots as long as the cumulative number of non conformities is no greater than one.
4. Once k_2 lots have been accepted, cumulative the number of non conformities only over the most recent k_2 lots, and continue to accept lots as long as this cumulative number of non conformities is no greater than one.
5. If, at any stage, the cumulative number of non conformities becomes greater than one, reject the current lot and return to step1.

The three parameters of a Chsp-(0, 1) plan are n, k_1, k_2, where

n - Sample size

k_1 - Minimum number of successive samples required to b free from non conformities before cumulation can takes place.

k_2 - Maximum number of successive samples over which the cumulation of non conformities takes place.The operating characteristic function $P_a(p)$ of Chsp-(0,1) plan with parameters n, k_1, and k_2 was derived by Dodge and Stephens(1966) as

$$P_a(p) = \frac{P_0(1-P_0) + P_1 P_0^{k_1}(1-P_0)^{k_2-k_1}}{1-P_0 + P_1 P_0^{k_1}(1-P_0^{k_2-k_1-1})} \quad k_2 > k_1$$

Where

P_0 = Probability of getting exactly zero non conformities in a sample of size n.

P_1 = Probability of getting exactly one non conformity in a sample of size n.

It is well known that for a series of lots from a process, the binomial model for the OC curve will be exact in the case of fraction non conforming. It can be satisfactorily approximated with the Poisson model when p is small, n is large. When the quality is measured in terms of non conformities, the Poisson model is approximate one.

Under the Poisson assumption, $P_0 = e^{-np}$ and $P_1 = npe^{-np}$

Selection Procedure

Designing of Quality Interval Chsp-(0, 1) plan as follows

Quality Decision Region (QDR)

It is an interval of quality ($p_1<p<p^*$) in which products is accepted at engineers quality average. This quality is reliably maintained up to p* (MAPD) and sudden decline in quality is expected. This region is also called Reliable Quality Region (RQR).

Quality Decision Range denoted as $d_1 = p^* - p_1$ is derived from probability of acceptance.

P_a (p_1<p<p*) = $$P_a(p) = \frac{P_0(1-P_0) + P_1 P_0^{k_1}(1-P_0)^{k_2-k_1}}{1-P_0 + P_1 P_0^{k_1}(1-P_0^{k_2-k_1-1})}$$, for p_1<p<p*

Where $P_0 = e^{-np}$ and $P_1 = npe^{-np}$

Probability Quality Region (PQR)

It is an interval of quality $(p_1 < p < p_2)$ in which product is accepted with a minimum probability 0.10 and maximum probability 0.95.

Probabilistic Quality Range denoted as $d_2 = p_2 - p_1$ is derived from probability of acceptance.

P_a (p_1<p<p_2) = $$P_a(p) = \frac{P_0(1-P_0) + P_1 P_0^{k_1}(1-P_0)^{k_2-k_1}}{1-P_0 + P_1 P_0^{k_1}(1-P_0^{k_2-k_1-1})}$$, for p_1<p<p_2

Where $P_0 = e^{-np}$ and $P_1 = npe^{-np}$

Construction of Tables

The probability of acceptance for ChSP-(0, 1) under Poisson model is given by

$$P_a(p) = \frac{P_0(1-P_0) + P_1 P_0^{k_1}(1-P_0)^{k_2-k_1}}{1-P_0 + P_1 P_0^{k_1}(1-P_0^{k_2-k_1-1})}$$

Where $P_0 = e^{-np}$ and $P_1 = npe^{-np}$

The inflexion point (p*) is obtained by using $\frac{d^2 P_a(p)}{dp^2} = 0$ and $\frac{d^3 P_a(p)}{dp^3} \neq 0$

$$\frac{dP_a(p)}{dp} = 2ne^{-2np} + 2ne^{-np(1+k_1)} - 2n^2 p(1+k_1)e^{-np(1+k_1)} - ne^{-(1+k_1)} - ne^{-npk_2} + n^2 pk_2 e^{-npk_2}$$

$$\frac{d^2 P_a(p)}{dp^2} = -4n^2 e^{-2np} - 4n^2(1+k_1)e^{-np(1+k_1)} + 2n^3 p(1+k_1)^2 e^{-np(1+k_1)} + 2n^2 k_2 e^{-npk_2} - n^3 pk_2{}^2 e^{-npk_2}$$

Equating to zero, then we get

$$-4n^2 e^{-2np} - 4n^2(1+k_1)e^{-np(1+k_1)} + 2n^3 p(1+k_1)^2 e^{-np(1+k_1)} + 2n^2 k_2 e^{-npk_2} - n^3 pk_2{}^2 e^{-npk_2} = 0$$

The relative slope of OC Curve h* = $\left[\frac{-p}{P_a(p)}\right]\frac{dP_a(p)}{dp}$ at p = p*. The inflexion tangent of the OC Curve cuts the p-axis at p_t =p*+(p*/h*). The value of h*, np_t, np* are calculated and presented in table 1.

Table 1: Certain characteristic values tabulated against k_1 and k_2 for ChSP-(0, 1)

k_1	k_2	np_1	np_2	np*	h*	np_t
1	2	0.206640	2.490239	0.310396	1.62345	0.50159
1	3	0.026854	2.460571	0.058050	1.155900	0.102700
1	4	0.017805	2.445630	0.038225	1.071600	0.073895
1	5	0.013401	2.432501	0.028883	0.712776	0.069404
1	6	0.010742	2.420056	0.023209	0.667564	0.057975
1	7	0.008963	2.408227	0.019395	0.621342	0.050609
1	8	0.007689	2.297036	0.016656	0.512435	0.049159

1	9	0.006732	2.386505	0.014595	0.442798	0.048910
1	10	0.005987	2.376646	0.012987	0.357234	0.045341
2	3	0.162158	2.324586	0.245280	2.989750	0.327320
2	4	0.027214	2.320379	0.059714	2.762321	0.319076
2	5	0.180201	2.318255	0.039236	2.729320	0.297689
2	6	0.013538	2.316491	0.029541	2.542900	0.287690
2	7	0.010837	2.314902	0.023665	2.232671	0.275610
2	8	0.009032	2.313461	0.019727	1.564329	0.266722
2	9	0.007741	2.312153	0.016909	1.237868	0.265020
2	10	0.006773	2.310967	0.014793	1.090565	0.264301
3	4	0.138910	2.304872	0.211675	3.723460	0.268524
3	5	0.027583	2.304417	0.061458	3.543438	0.253421
3	6	0.018242	2.304190	0.040315	3.216795	0.2465423
3	7	0.013680	2.304003	0.030240	2.743400	0.2348760
3	8	0.010934	2.303836	0.024146	2.6550919	0.233100
3	9	0.009102	2.303686	0.020077	2.434767	0.231090
3	10	0.007794	2.303551	0.017173	2.315690	0.225970
4	5	0.124125	2.302815	0.190545	6.564320	0.219572
4	6	0.027961	2.302769	0.063283	6.265443	0.2154300
4	7	0.018470	2.302746	0.041469	6.165800	0.2144870
4	8	0.013826	2.302727	0.030986	5.823400	0.2121900
4	9	0.011033	2.302711	0.024656	5.784321	0.205179

4	10	0.009174	2.302695	0.020444	5.760223	0.203217
5	6	0.113702	2.302608	0.175812	5.432980	0.208172
5	7	0.028348	2.302603	0.065185	5.254797	0.207120
5	8	0.018706	2.302601	0.042706	5.228704	0.205671
5	9	0.013976	2.302599	0.031782	5.126589	0.205211
5	10	0.011135	2.302598	0.025197	4.656408	0.203200
6	7	0.105868	2.302587	0.164864	7.543200	0.202540
6	8	0.028745	2.302587	0.067159	7.354311	0.201540
6	9	0.018949	2.302587	0.044034	6.245980	0.201021
6	10	0.014130	2.302587	0.032635	6.084300	0.200000
7	8	0.099718	2.302585	0.156371	5.435409	0.185139
7	9	0.029151	2.302585	0.069197	5.324500	0.1845300
7	10	0.019200	2.302585	0.045626	5.301230	0.1834590
8	9	0.094734	2.302585	0.149573	4.897800	0.181638
8	10	0.029567	2.302585	0.071287	4.567890	0.180120
9	10	0.090598	2.302585	0.144006	4.734778	0.17047

CONCLUSION

We introduce a new procedure for construction and selection of ChSP-(0, 1) plan using quality region and conversion of parameters are provided for comparison. To ensure higher standard of quality attainment for product or process, Quality Interval Sampling is a very good measure for defining quality and designing and acceptance sampling plan. The MAPD is a key measure to find out what degree the inflexion point empowers the OC Curve to discriminate between good and bad lots.

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