Nano Generalized δ Semi -Regular and Normal Spaces

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Abstract

The basic objective of this paper is to introduce and investigate the properties of few Ngδs-regular and Ngδs-normal spaces in nano topological spaces and studied some of its relation between the existing sets.

1. Introduction

In 1970, Levine introduced the concept of generalized closed sets in topological spaces. The notion of Nano topology was introduced by Lellis Thivagar which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure of a set. Sakthivel and Devaki introduced nano generalized δ semi-closed sets in nano topological spaces. The basic definitions are recalled from the following papers [1],[2],[3],[4],[5],[6],[7],[8].

2. Nano generalized δ semi -regular spaces

Definition 2.1. A topological space U is said to be Ngδs-regular if for each nano closed set F and each point $x \in U - F$, there exist disjoint Ngδs-open sets P and Q such that $x \in P$ and $F \subset Q$.

Remark 2.2. Every nano regular space is Ngδs-regular space, but converse need not be true.

Example 2.3. Let U = {a,b,c,d} and U/R = {{a,d},{b},{c}}, X = {a,b}.

$\tau_R(X)$ = {U,φ,{b},{a,b,d},{a,d}}and $\tau_R c(X)$ = {U,φ,{c},{a,c,d},{b,c}} be a nano topology on U then Ngδs-closed = {{a},{b},{c},{d},{a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d},U,φ}

Let F = {b} and x = {a} there P = {a,d} and Q = {b,c} is nano Ngδs-regular space but not nano regular space.

Theorem 2.4. Every Ngδs-regular T_0 space is Ngδs-T_2.

Proof: Let U be a Ngδs-regular space and $x,y \in U$ such that $x \neq y$. Since U being T_0 space, there exists an nano-open set Q in U such that $x \in Q$ and $y \notin Q$. Then $U - Q$ is a nano –closed set containing y but not x. Since U is Ngδs regular, there exist disjoint Ngδs-open sets P and

R such that $U - Q \subset R$ and $x \in P$. Now $y \in U - Q$ implies $y \in W$. Thus for $x, y \in U$ such that $x \neq y$, there exist disjoint nano open sets P and R such that $x \in P$ and $y \in R$. Therefore U is Ngδs-T_2 space.

Theorem 2.5. In a nano topological space U, the following conditions are equivalent.

(i) U is Ngδs-regular.

(ii) For every point $x \in U$ and nano open set Q containing x there exists a Ngδs-open set P such that $x \in P \subset$ Ngδs-cl(P) $\subset Q$.

(iii) For every nano closed set F, $F = \cap\{$Ngδs-cl(Q) $: F \subset Q$ and Q is Ngδs-open set of U$\}$.

(iv) For every set A and an nano open set B such that $A \cap B \neq \emptyset$, there exists Ngδs-open set O such that $A \cap O \neq \emptyset$ and Ngδs-cl(O) $\subset B$.

(v) For every non empty set A and nano closed set B such that $A \cap B \neq \emptyset$, there exist disjoint Ngδs-open sets L and M such that $A \cap L \neq \emptyset$ and $B \subset M$.

Proof: (i) ⇒ (ii) Let Q be an nano open set containing x. Then $U - Q$ is nano closed set not containing x. Since U is Ngδs-regular, there exist Ngδs-open sets L and P such that $x \in P$, $U - Q \subset L$ and $P \cap L = \emptyset$. This implies $P \subset U - L$.

Therefore, Ngδs-cl(P) $\subset$ Ngδs-cl($U - L$) $= U - L$, because $U - L$ is Ngδs-closed.

Hence $x \in P \subset$ Ngδs-cl(P) $\subset U - L \subset Q$. That is $x \in P \subset$ Ngδs-cl(P) $\subset Q$.

(ii) ⇒ (iii) Let F be a nano closed set and $x \notin F$. Then $U - F$ is an nano open set containing x. By (ii), there is a Ngδs-open set P such that $x \in P \subset$ Ngδs-cl(P) $\subset U - F$. And so, $F \subset U -$Ngδs-cl(P) $\subset U - P$. Consequently $U - P$ is Ngδs-closed set not containing x.

Put $Q = U -$Ngδs-cl(P). This implies $F \subset Q$ and QF is Ngδs-open set of U and $x \notin$ Ngδs-cl(Q), implies $\cap\{$Ngδs-cl(Q) $: F \subset Q$ and Q is Ngδs-open set of U$\} \subset F$.......(1) But is nano closed and every nano closed set is Ngδs-closed. Therefore $F \subset \cap$Ngδs-cl(Q) $: F \subset Q$ and Q is Ngδs-open set of U$\}$....... (2) is always true. From (1) and (2), $F = \cap\{$Ngδs-cl(Q)$: F \subset Q$ and Q is Ngδs-open set of U$\}$.

(iii)⇒ (iv) Let $A \cap B \neq \emptyset$ and B is nano open. Let $x \in A \cap B$. Then $U - B$ is a nano closed set not containing x. By (iii), there exists a Ngδs-open set Q of U such that $U - B \subset Q$ and $x \notin$ Ngδs-cl(Q). Put $O = U -$Ngδs-cl(Q), then O is Ngδs-open set of U, $x \in A \cap O$ and Ngδs-cl(O) $\subset$ Ngδs-cl($U - Q$) $= U - Q \subset B$. Hence Ngδs-cl(O) $\subset B$.

(iv) ⇒ (v) If $A \cap B \neq \emptyset$, where A is non empty and B is nano closed, then $A \cap (U - B) \neq \emptyset$ and $(U - B)$ is nano open. Therefore by (iv), there exists Ngδs-open set L such that $A \cap L \neq \emptyset$ and $L \subset$ Ngδs-cl(L) $\subset U - B$.

(v) ⇒ (i) Let F be a nano closed set such that $x \notin F$, then $\{x\} \cap F = \emptyset$. By (v), there exist disjoint nano open sets L and M such that $\{x\} \cap L \neq \emptyset$ and $F \subset M$, which implies $x \in L$ and

F ⊂ M. Hence, U is Ngδs-regular.

Theorem 2.6. If f :U → V is nano continuous bijective, Ngδs-open (resp. NPgδs-open) function and U is a nano regular (resp. NS-regular) space, then V is Ngδs-regular.

Proof: Let F be a nano closed set in V and y ∉ F. Take y = f(x) for some x ∈ U. Since f is nano continuous $f^{-1}(F)$ is nano closed set in U such that x ∉ $f^{-1}(F)$. Now U is nano regular (resp. NS-regular), there exist disjoint nano open (resp. NS-open) sets P and Q such that x ∈ P and $f^{-1}(F)$ ⊂ Q. That is, y = f(x) ∈ f(P) and F ⊂ f(Y). Since f is Ngδs-open (resp. NPgδs-open) function f(P) and f(Y) are Ngδs-open sets in V and f is bijective f(P) ∩ f(Q) = f (P ∩ Q) = f(∅) = ∅. Therefore, V is Ngδs-regular.

Theorem 2.7. If f :U → V is nano semi continuous bijective, NPgδs-open function and U is nano semi-regular space, then V is Ngδs-regular.

Proof: Let F be a nano closed set in V and y ∉ F. Take y = f(x) for some x ∈ U. Since f is nano semi continuous $f^{-1}(F)$ is nano semi closed set in U and x ∉ $f^{-1}(F)$. Now U is nano semi regular, there exist disjoint nano semi-open sets P and Q such that x ∈ P and $f^{-1}(F)$⊂ Q. That is, y = f(x) ∈ f(P) and F ⊂ f(Q). Since f is NPgδs-open function f(P) and f(Q) are Ngδs-open sets in V and f is bijective f(P) ∩ f(Q) = f(P ∩ Q) = f(∅) = ∅. Therefore, V is Ngδs-regular.

Theorem 2.8. If f :U → V is nano continuous surjective, strongly Ngδs-open (resp. quasi Ngδs-open) function and U is Ngδs-regular space, then V is Ngδs-regular (resp. nano-regular).

Proof: Let F be a nano closed set in V and y ∉ F. Take y = f(x) for some x ∈ U. Since f is nano continuous surjective $f^{-1}(F)$ is nano closed set in U and x ∉ $f^{-1}(F)$. Now since U is Ngδs-regular, there exist disjoint Ngδs-open sets P and Q such that x ∈ P and $f^{-1}(F)$ ⊂ Q. That is, y = f(x) ∈ f(P) and F ⊂ f(Q). Since f is strongly Ngδs-open (resp. quasi Ngδs-open) and bijective, f(P) and f(Q) are disjoint Ngδs-open (resp. nano-open) sets in V . Therefore, V is Ngδs-regular (resp. nano-regular).

Theorem 2.9. If f :U → V is Ngδs-continuous, nano-closed, injection and V is nano-regular, then U is Ngδs-regular.

Proof: Let F be a nano closed set in U and x ∉ F. Since f is nano closed injection f(F) is nano closed set in V such that f(x) ∉ f(F). Now V is nano-regular, there exist disjoint nano-open sets G and H such that f(x) ∈ G and f(F) ⊂ H. This implies x ∈ $f^{-1}(G)$ and F ⊂ $f^{-1}(H)$. Since f is Ngδs-continuous,$f^{-1}(G)$ and $f^{-1}(H)$ are NgδsNS-open sets in U. Further $f^{-1}(G) \cap f^{-1}(H)$ = ∅. Hence U is Ngδs-regular.

Theorem 2.10. If f :U → V is nano semi Ngδs-continuous, nano-closed (resp. NS-closed), injection and V is NS-regular (resp. NS-regular) then U is Ngδs-regular.

Proof: Let F be a nano-closed set in U and x ∉ F. Since f is nano-closed (resp. NS-closed) injection f(F) is nano-closed (resp. NS-closed) set in V such that f(x) ∉ f(F). Now V is N-regular (resp. NS-regular), there exist disjoint nano semi-open sets G and H such that f(x) ∈ G and f(F) ⊂ H. This implies x ∈ f^{-1}(G) and F ⊂ f^{-1}(H) Since f is semi Ngδs-continuous f^{-1}(G) and f^{-1}(H) are Ngδs-open sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence U is Ngδs-regular.

Theorem 2.11. If f :U→V is strongly Ngδs-continuous, nano-closed, injection and V is Ngδs-regular then U is NS-regular.

Proof: Let F be a nano-closed set in U and x ∉ F. Since f is nano-closed injection f(F) is nano-closed set in V such that f(x) ∉ f(F). Now V is Ngδs-regular, there exist disjoint Ngδs-open sets G and H such that f(x) ∈ G and f(F) ⊂ H. This implies x ∈ f^{-1}(G) and F ⊂ f^{-1}(H). Since f is strongly Ngδs-continuous f^{-1}(G) and f^{-1}(H) are nano-open sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence U is nano-regular (resp. NS-regular).

Theorem 2.12. If f :U → V is Ngδs-irresolute, nano-closed, injection and V is NS-regular then U is Ngδs-regular.

Proof: Let F be a nano-closed set in U and x ∉ F. Since f is nano closed injection f(F) is nano-closed set in V such that f(x) ∉ f(F). Now V is Ngδs-regular, there exist disjoint Ngδs-open sets G and H such that f(x) ∈ G and f(F) ⊂ H. This implies x ∈ f^{-1}(G) and F ⊂ f^{-1}(H). Since f is Ngδs-irresolute f^{-1}(G) and f^{-1}(H) are Ngδs-open sets in U.
Further $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Hence U is Ngδs-regular.

Corollary 2.13. If f :U → V is perfectly Ngδs-continuous injective, nano-closed function and V is NS-regular space, then U is regular.

Proof: Obvious, because every perfectly Ngδs-continuous is strongly Ngδs-continuous.

Theorem 2.14. If f :U → V is perfectly Ngδs-continuous injective, nano-closed function and V is Ngδs-regular space, then U is ultra regular.

Proof: Let F be a nano-closed set in U and x ∉ F. Since f is nano-closed injective, f(F) is nano-closed in V such that f(x) ∉ f(F). By Ngδs-regularity of V , there exist disjoint Ngδs-open sets P and Q such that f(x) ∈ P and f(F) ⊂ Q. That is x ∈ f^{-1}(P) and F ⊂ f^{-1}(Q). Since f is perfectly Ngδs-continuous f^{-1}(P) and f^{-1}(Q) are disjoint nano-clopen sets in U. Therefore, U is ultra nano-regular.

Theorem 2.15. If f :U → V is totally nano-continuous bijective, Ngδs-open function and U is nano-clopen-regular space, then V is Ngδs-regular.

Proof: Let F be a nano-closed set in V and y ∉ F. Take y = f(x) for some x ∈ U. Since f is totally nano-continuous bijective, $f^{-1}(F)$ is nano clopen in U such that x ∉ $f^{-1}(F)$. By nano clopen-regularity of U, there exist disjoint nano-open sets P and Q such that x ∈ P and $f^{-1}(F) \subset Q$. That is y = f(x) ∈ f(P) and F ⊂ f(Q). Since f is Ngδs-open bijective f(P) and f(Q) are disjoint Ngδs-open sets in V . Therefore, V is Ngδs-regular.

Theorem 2.16. If f :U → V is completely nano-continuous injective, Ngδs-open function and U is almost nano regular space, then V is Ngδs-regular.

Proof: Let F be a nano-closed set in V and y ∉ F. Take y = f(x) for some x ∈ U. Since f is completely nano continuous, $f^{-1}(F)$ is nano regular open in U such that x ∉ $f^{-1}(F)$. By almost nano regularity of U, there exist disjoint nano-open sets P and Q such that x ∈ P and $f^{-1}(F) \subset Q$. That is y = f(x) ∈ f(P) and F ⊂ f(Q). Since f is Ngδs-open and injective, f(P)and f(Q) are disjoint Ngδs-open sets in V . Therefore, V is Ngδs-regular.

3. Nano generalized δ semi -normal spaces

Definition 3.1. A nano topological space A is said to be Ngδs-normal if for every pair of disjoint nano-closed sets E and F of U there exist disjoint Ngδs-open sets P and Q such that E ⊂ P and F ⊂ Q.

Remark 3.2. Every nano normal space is Ngδs-normal space, but converse need not be true.

Theorem 3.3. The following statements are equivalent for a nano topological space U.

1. U is Ngδs-normal.
2. For each nano closed set A and for each nano-open set P containing A, there exists a Ngδs - open set Q containing A such that Ngδs-cl(Q) ⊂ P.
3. For each pair of disjoint nano-closed sets A and B there exists a Ngδs-open set P containing A such that Ngδs-cl(P) ∩ B = ∅.

Proof: (i) ⇒ (ii) Let A be nano-closed set and P be an nano-open set containing A. Then A ∩ (U −P) = ∅, therefore they are disjoint nano-closed sets in U. Since U is Ngδs-normal their exist disjoint Ngδs-open sets Q and W such that A ⊂ Q, U −P ⊂ W that is U −W ⊂ P. Now Q ∩ W = ∅, implies Q ⊂ U −W. Therefore Ngδs-cl(Q) ⊂ Ngδs-cl(U −W) = U −W, because U −W is Ngδs-closed set. Thus, A ⊂ Q ⊂ Ngδs-cl(Q) ⊂ U −W ⊂ P.

That is A ⊂ Q ⊂ Ngδs-cl(Q) ⊂ P.

(ii) ⇒ (iii) Let A and B be disjoint nano closed sets in A, then A ⊂ U −B and X −B is an nano-open set containing A. By (ii), there exists a Ngδs-open set P such that A ⊂ P and Ngδs-cl(P) ⊂ U −B, which implies Ngδs-cl(P) ∩ B = ∅.

(iii) ⇒ (i) Let A and B be disjoint nano-closed sets in A. By (iii) there exists a Ngδs-open set P such that A ⊂ P and Ngδs-cl(P)∩B = ∅ or B ⊂ U −Ngδs-cl(P). Now P and U −Ngδs-cl(P) are disjoint Ngδs-open sets of U such that A ⊂ P and B ⊂ U −Ngδs-cl(P). Hence U is Ngδs-normal.

Theorem 3.4. If U is nano-semi normal and F ∩ A = ∅, where F is Nδ-closed set and A is Ngδs-closed, then there exist disjoint nano semi-open sets P and Q in U such that F ⊂ P and A ⊂ Q.

Proof: Let U be a nano semi normal space and F ∩A = ∅, where F is Nδ-closed set and A is Ngδs-closed set. Then A ⊂ U−F and U−F is Nδ-open set. Therefore NS-cl(A) ⊂ U −F, implies NS-cl(A) ∩ F = ∅. Now F is Nδ-closed hence nano semi-closed, which implies F and NS-cl(A) are disjoint nano semi-closed sets and U is nano semi-normal, therefore there exist disjoint nano semi-open sets P and Q of U such that F ⊂ P and NS-cl(A) ⊂ Q. That is, P and Q of U are such that F ⊂ P and A ⊂ Q.

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