$*_s$–Connectedness In Fuzzy Ideal Topological Spaces

K.Priyadharsini,

Assistant Professor,

Department Of Mathematics,

SNMV CAS,Coimbatore.

ABSTRACT

In this paper the new concepts of fuzzy *-connectedness, fuzzy *-Hyper connectedness, fuzzy *-separated are introduced and explained with examples.

Key words: fuzzy *-connectedness,fuzzy *-Hyper connectedness, fuzzy *-separated

1.Introduction

The topic of ideals in general topological spaces is treated in the classic text by K.Kuratowski[2]. The notion of connectedness and *-connectedness in ideal topological spaces are introduced and studied by Erdal Ekici, Takashi Noiri[1]. In this paper attempt's to generalize there concepts in fuzzy ideal topological spaces. Examples are also included to reinforce the concepts, and to point out that the ideas are present in similar areas with which the reader may be familiar with.

2. $*_S$–Connectedness In Fuzzy Ideal Topological Spaces

Definition : 2.1

Nonempty fuzzy subsets A, B of a fuzzy ideal space $(X,\tau,\mathscr{I})$ are called

fuzzy $*$ - separated if $Cl^*(A)\wedge B=A\wedge Cl(B) = \bar{0}$

Example : 2.2

(i) Let X be any set with more than 2 points

$\mathscr{I}= I^x ;\tau = I^x$

Then $(X,\tau,\mathscr{I})$ is a fuzzy ideal topological space.

$\tau^* = I^x$

$\therefore$ Every A in I^x is fuzzy open, fuzzy closed, fuzzy $*$ - open and fuzzy $*$ - closed.

Let $A = p_1$; $B = q_1$ be the fuzzy points

$Cl^* (A) = p_1$; $Cl (B) = q_1$

$Cl^{*}(A)\Lambda B = p_1 \wedge q_1 = \bar{0}$

$A \wedge Cl(B) = p_1 \wedge q_1 = \bar{0}$

$\therefore$ A and B are fuzzy $*$ - separated

(ii) Let $A = p_1$; $B = p_1 \wedge q_1$

$Cl^{*}(A) = p_1$; $Cl(B) = p_1 \wedge q_1$

$Cl^{*}(A) \wedge B = p_1 \wedge (p_1 \vee q_1) \neq \bar{0}$

[since $p_1 \wedge (p_1 \vee q_1)$] (p) = min (1,1) = 1]

$\therefore$ Here A and B are not fuzzy $*$ - separated.

Definition: 2.3

A fuzzy subset A of an ideal space $(X,\tau,\mathscr{I})$ is called fuzzy $*_s$ - connected if A is not the fuzzy union of two fuzzy $*$ - separated sets in $(X,\tau,\mathscr{I})$.

Theorem : 2.4

Let Y be a fuzzy open subset of a ideal space $(X,\tau,\mathscr{I})$. The following are equivalent.

1. Y is fuzzy $*_s$ – connected in $(X,\tau,\mathscr{I})$
2. Y is fuzzy $*$ - connected in $(X,\tau,\mathscr{I})$

Proof :

(1)$\Rightarrow$ (2)

Suppose Y is not fuzzy $*$ - connected Then there exists nonempty disjoint fuzzy open and fuzzy $*$ - open sets A, B in Y $\ni$ $Y = A \vee B$

Since Y is fuzzy open in X

A and B are fuzzy open & fuzzy $*$ - open respectively in X.

$Y = A \vee B$ and $A \wedge B = \bar{0}$

$\Rightarrow B = A^c$ in Y

A is fuzzy open in X

$\Rightarrow$ B is fuzzy closed in X

On the other hand

$A = B^c$ in Y

B is fuzzy $*$ - open in X

$\Rightarrow$ A is fuzzy $*$ - closed in X

$\therefore \bar{0} = A \wedge B$

$= Cl^{*}(A) \wedge B = A \wedge Cl(B)$

$\therefore$ A&B are fuzzy $*$ - separated in X.

This implies Y is not fuzzy $*_s$ connected in X.

which is a contradiction

$\therefore$ Y is fuzzy $*$ - connected in X.

(2) ⇒ (1)

If Y is not $*_s$ – connected in X. Then there exists a fuzzy $*$ - separated sets A&B

$\ni$ Y = A$\vee$B.

A&B are fuzzy open & fuzzy $*$ - open in X respectively.

A, B are fuzzy open & fuzzy $*$ open in Y respectively.

Since A&B are fuzzy $*$ - separated

$Cl^*(A)\wedge B = \bar{0} = A\wedge Cl(B)$

$\therefore A\wedge B = \bar{0}$

$\therefore$ A, B are disjoint

This implies Y = A$\vee$B

⇒ Y is not fuzzy $*$ - connected in X

which is a contradiction

$\therefore$ Y is fuzzy $*_s$ connected in X.

Theorem : 2.5

Let $(X,\tau,\mathscr{I})$ be a fuzzy ideal space. If A is a fuzzy $*_s$ connected set of X and H,G are fuzzy $*$ - separated sets of X with $A \le H\vee G$, then either $A \le H$ or $A \le G$.

Proof :

Let $A \le H\vee G$

$(A\wedge H) \vee (A\wedge G)=A\wedge(H\vee G)$

$=A$

Then $(A\wedge G) \wedge Cl^*(A\wedge H) \le G\wedge Cl^*(H) = \bar{0}$

Similarly $(A\wedge H) \vee Cl(A\wedge G) = \bar{0}$

$\therefore$ A$\wedge$H & A$\wedge$G all fuzzy $*$ - separated

But A = (A$\wedge$H) $\vee$ (A$\wedge$G) ---------------------- (1)

If A$\wedge$H & A$\wedge$G are nonempty

(1) ⇒ A is not fuzzy $*_s$ – connected

which is a contradiction

$\therefore$ Either A$\wedge$H = 0 (or) A$\wedge$G = 0

This ⇒ $A \le H$ (or) $A \le G$.

Theorem : 2.6

If A is fuzzy $*_s$ – connected set of an ideal space $(X,\tau,\mathcal{I})$ and

$A \leq B \leq Cl^*(A)$, then B is fuzzy $*_s$ – connected.

Proof :

Suppose B is not fuzzy $*_s$ – connected.

Then there exists fuzzy $*$ - separated sets H & G

Such that $B = H \vee G$ ------------------- (1)

This implies that $H \neq \bar{0}$ & $G \neq \bar{0}$ and

$G \wedge Cl^*(H) = \bar{0} = H \wedge Cl(G)$

Either $A \leq H$ or $A \leq G$

Suppose that $A \leq H$

Then $Cl^*(A) \leq Cl^*(H)$

But $G \wedge Cl^*(H) = \bar{0}$

$\therefore G \wedge Cl^*(A) = \bar{0}$

(1)$\Rightarrow$ $G \leq B \leq Cl^*(A)$

$\therefore G = G \wedge Cl^*(A) = \bar{0}$

Which is a contradiction

Suppose that $A \leq G$

Then $Cl(A) \leq Cl(G)$

But $H \wedge Cl(G) = \bar{0}$

$\therefore H \wedge Cl(A) = \bar{0}$

(1)$\Rightarrow$ $H \leq B \leq Cl^*(A) \leq Cl(A)$

$\therefore H = H \wedge Cl(A) = \bar{0}$

Which is a contradiction

$\therefore$ B is fuzzy $*_s$ – connected

Corollary : 2.7

If A is a fuzzy $*_s$ – connected set in an ideal space $(X,\tau,\mathcal{I})$ then $Cl^*(A)$ is fuzzy $*_s$ – connected.

Proof :

In the previous theorem put $B = Cl^*(A)$

$\therefore$ By previous theorem $Cl^*(A)$ is fuzzy $*_s$ – connected

References

1. Erdal Ekici., Takashi Noiri.,Connectedness in ideal topological spaces.NOVI SAD J.Math .Vol.38,NO. 2,65-70.
2. Kuratowski, K. Topology ,Vol.I.Newyork: Academic Press,1966.
3. Levine, N.,Am, Math. Monthly 70 (1) (1963),36-41.
4. Levine, N., Am, Math. Monthly 75 (1968),847-52.
5. Maheswari, S.N., and Tapi,U., Nanta.Math 12(1979),102-109.
6. Mathew. P. M., On Hyperconnected spaces,Indian J.Pure appl.Math.19(12):(1998),1180-1184 .