Ascending Pendant Domination Decomposition of Special Graphs

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Abstract:

Let $G = (V, E)$ be a simple connected graph. We have introduced Ascending Pendant Domination Decomposition of Graphs **[2]** and defined by APDD of a graph G is a collection $\{G_1, G_2, G_3, \dots, G_n\}$ of subgraphs of G such that every edge of G is exactly once in G_i, each G_i is connected and $\gamma_{pe}(G_i) = i + 1, 1 \leq i \leq n$. In this paper, we obtain the special graphs namely $P_{\frac{n(n+3)}{2}} \odot K_1$, $C_{\frac{n(n+3)}{2}} \odot K_1$, $G_{4n(n-2)}, W_{(n+3)(n-1)}$ and $H_{\left(\frac{(n-1)(n+4)}{2}\right)}$ admits Ascending Pendant Domination Decomposition.

Keywords: Dominating set, Pendant Dominating set, Decomposition and Pendant Domination Decomposition

AMS Subject Classification: $05C69$ and $05C70$

1.Introduction

Let $G = (V, E)$ be a simple connected graph. All the graphs considered as finite and undirected. A vertex of degree zero is called an isolated vertex and a vertex of degree one is called a pendant vertex. An edge incident with a pendant vertex is called a pendant edge. Pendant Domination in some Generalised Graphs was introduced by Nayaka S.R Puttaswamy and S.Purushothama**[6]**. Ascending Domination Decomposition of Subdivision of Graphs was introduced by K. Lakshmiprabha and K. Nagarajan**[4]**.In this paper, we investigate the special class of graphs which are admits Ascending Pendant Domination Decomposition.

Definition 1.1.

If $G_1, G_2, G_3, \dots, G_n$ are connected edge disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \dots \cup E(G_n)$, then $(G_1, G_2, G_3, \dots, G_n)$ is said to be decomposition of G.

Definition 1.2.

A subset S of vertices in a graph G is called a Dominating set if every vertex $v \in V$ is either in S or adjacent to some vertex in S. The least cardinality of a dominating set in G is called the domination number of G and is usually denoted by $\gamma(G)$.

Definition 1.3.

A Dominating set S in G is called a Pendant Dominating set if $< S >$ contains atleast one pendant vertex. The minimum cardinality of a Pendant Dominating set is called the pendant domination number denoted by $\gamma_{pe}(G)$.

Definition 1.4.

The corona $G_1 \odot G_2$ of two graphs $G_1$and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has $p_1$vertices) and $p_1$copies of G_2 and then joining the i^{th} vertex of $G_1$to every vertex in the i^{th} copy of G_2.

Definition 1.5.

The graph $C_n \odot K_1$is called a Crown. The graph $P_n \odot K_1$ is called a comb.

Definition 1.6.

A wheel graph W_n is a cycle graph C_n with an additional central vertex adjacent to all the vertices on the cycle graph.

Definition 1.7.

The Gear graph G_n is obtained from a wheel W_n by adding as vertex between every pair of adjacent vertices of the $n-$ cycle of W_n.

Definition 1.8.

The Helm graph H_n is the graph obtained from an wheel graph by adjoining a pendant edge at each vertex of the cycle.

Definition 1.9.

For a vertex $v \in V$, the open neighbourhood of v is the set $N(v)$ containing all the vertices u adjacent to v and the closed neighbourhood of v is the set $N[v]$ containing v and all the vertices u adjacent to v.

Definition 1.10.[2]

A Decomposition $(G_1, G_2, \ldots, G_n)$ of G is said to be Ascending Pendant Domination Decomposition (APDD) if

(i) Each G_i is connected

(ii) $\gamma_{pe}(G_i) = i + 1, 1 \leq i \leq n.$

2.Main Results

Theorem 2.1

The comb $P_{\frac{n(n+3)}{2}} \odot K_1$ admits APDD into $n-$parts and $\gamma_{pe}(P_{\frac{n(n+3)}{2}} \odot K_1) = \sum_{i=1}^{n} \gamma_{pe}(G_i)$.

Proof.

Let $\{u_1, u_2, \dots, u_{\frac{n(n+3)}{2}}\}$ and $\{v_1, v_2, \dots, v_{\frac{n(n+3)}{2}}\}$ be the vertex set of $P_{\frac{n(n+3)}{2}} \odot K_1$.

Here, $\{v_1, v_2, \dots, v_{\frac{n(n+3)}{2}}\}$ be the pendant vertex set of $P_{\frac{n(n+3)}{2}} \odot K_1$.

Define $G_1 = \langle N[u_1] \rangle \cup (u_2, v_2)$

$G_2 = \langle N[u_3, u_4] \rangle \cup (u_5, v_5)$

$G_3 = \langle N[u_6, u_7, u_8] \rangle \cup (u_9, v_9)$

… … … … … … … … … … … … … … … … …

$G_n = \langle N[u_r, u_{r+1}, \dots, u_{r+n-1}] \rangle \cup (u_{r+n}, v_{r+n})$ where r can be calculated by using Newton's Divided Difference Formula.

To find : r

n	r	Δr	$\Delta^2 r$	$\Delta^3 r$	$\Delta^4 r$
1	1				
		2			
2	3		1		
		3		0	
3	6		1		0
		4		0	
4	10		1		
		5			
5	15				

$$n = n_0 + xh$$

$$n = 1 + x(1) \Rightarrow x = n - 1.$$

$$r = r_0 + x\frac{\Delta r_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 r_0 + \cdots$$

$$= 1 + (n-1)2 + \frac{(n-1)(n-2)}{2}1$$

$$r = \frac{n(n+1)}{2}$$

Here, $r + n - 1 = \frac{n^2+3n-2}{2}$ and $r + n = \frac{n(n+3)}{2}$.

Clearly, comb $P_{\frac{n(n+3)}{2}} \odot K_1$ can be decompose into $G_1, G_2, \ldots, G_n$ and $\gamma_{pe}(G_i) = i + 1, 1 \leq i \leq n$.

Hence the comb $P_{\frac{n(n+3)}{2}} \odot K_1$ admits APPD $(G_1, G_2, \ldots, G_n)$.

Also, the minimal pendant dominating set in $P_{\frac{n(n+3)}{2}} \odot K_1$ is $\{u_1, u_2, \ldots, u_{\frac{n(n+3)}{2}}\}$.

Therefore, $\gamma_{pe}(P_{\frac{n(n+3)}{2}} \odot K_1) = \frac{n(n+3)}{2}$.

Now,

$$\sum_{i=1}^{n} \gamma_{pe}(G_i) = \gamma_{pe}(G_1) + \gamma_{pe}(G_2) + \cdots + \gamma_{pe}(G_n)$$

$$= 2 + 3 + \cdots + n + 1$$

$$= (1 + 2 + 3 + \cdots + n + n + 1) - 1$$

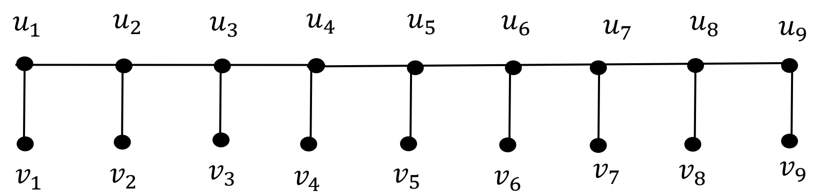
$$= \frac{(n+1)(n+2)}{2} - 1$$

$$= \frac{n(n+3)}{2}.$$

Hence $\gamma_{pe}(P_{\frac{n(n+3)}{2}} \odot K_1) = \sum_{i=1}^{n} \gamma_{pe}(G_i)$.

Illustration 2.2

In Figure 1, For $n = 3$, $P_9 \odot K_1$ admits APDD into 3 −parts



$P_9 \odot K_1$

APDD of $P_9 \odot K_1$

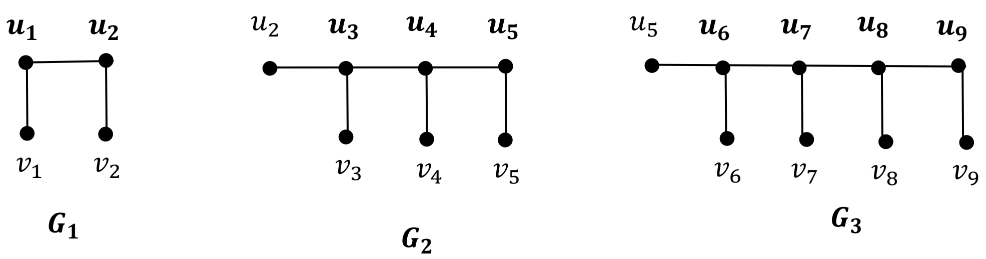


Figure 1

Here $\gamma_{pe}(G_1) = 2$, $\gamma_{pe}(G_2) = 3$ and $\gamma_{pe}(G_3) = 4$.

Theorem 2.3.

The Crown $C_{\frac{n(n+3)}{2}} \odot K_1$ admits APDD into n −parts and $\gamma_{pe}(C_{\frac{n(n+3)}{2}} \odot K_1) = \sum_{i=1}^{n} \gamma_{pe}(G_i) + 1$.

Proof.

Let $\{u_1, u_2, \dots, u_{\frac{n(n+3)}{2}}\}$ and $\{v_1, v_2, \dots, v_{\frac{n(n+3)}{2}}\}$ be the vertex set of $C_{\frac{n(n+3)}{2}} \odot K_1$.

Here, $\{v_1, v_2, \dots, v_{\frac{n(n+3)}{2}}\}$ be the pendant vertex set of $C_{\frac{n(n+3)}{2}} \odot K_1$.

Define $G_1 = \langle N[u_2] \rangle \cup (u_1, v_1)$

$G_2 = \langle N[u_4, u_5] \rangle \cup (u_3, v_3)$

$G_3 = \langle N[u_7, u_8, u_9] \rangle \cup (u_6, v_6)$

… … … … … … … … … … … … … … … … …

$G_n = \langle N[u_{r+1}, u_{r+2}, \dots, u_{r+n}] \rangle \cup (u_r, v_r)$ where r can be calculated by using Newton's Divided Difference Formula.

To find : r

n	r	Δr	$\Delta^2 r$	$\Delta^3 r$	$\Delta^4 r$
1	1				
		2			
2	3		1		
		3		0	
3	6		1		0
		4		0	
4	10		1		
		5			
5	15				

$$n = n_0 + xh$$

$$n = 1 + x(1) \Rightarrow x = n - 1.$$

$$r = r_0 + x\frac{\Delta r_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 r_0 + \cdots$$

$$= 1 + (n-1)2 + \frac{(n-1)(n-2)}{2}1$$

$$r = \frac{n(n+1)}{2}.$$

Here, $r + n = \frac{n(n+3)}{2}$.

Clearly, The crown $C_{\frac{n(n+3)}{2}} \odot K_1$ can be decompose into $G_1, G_2, \dots, G_n$ and $\gamma_{pe}(G_i) = i + 1, 1 \le i \le n$.

Hence the crown $C_{\frac{n(n+3)}{2}} \odot K_1$ admits APDD $(G_1, G_2, \dots, G_n)$.

Also, the minimal pendant dominating set in $C_{\frac{n(n+3)}{2}} \odot K_1$ is $\{u_1, u_2, \dots, u_{\frac{n(n+3)}{2}}\}$ and any one vertex in the vertex set $\{v_1, v_2, \dots, v_{\frac{n(n+3)}{2}}\}$.

Therefore, $\gamma_{pe}\left(C_{\frac{n(n+3)}{2}} \odot K_1\right) = \frac{n(n+3)}{2} + 1 = \frac{(n+1)(n+2)}{2}$.

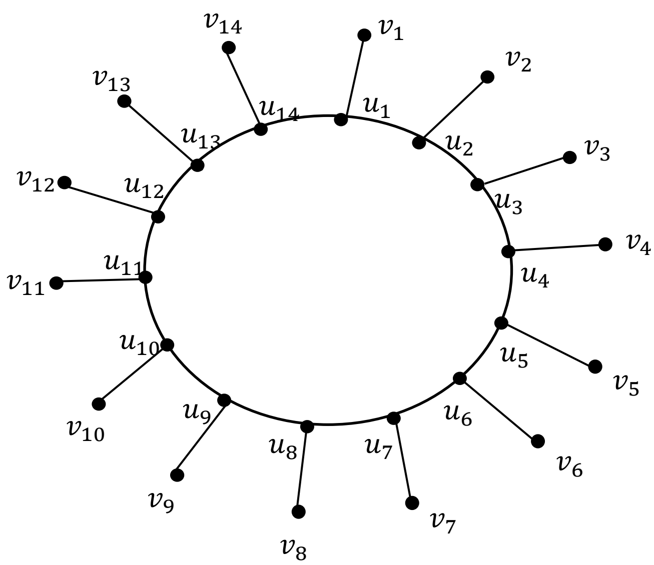
Now, $\sum_{i=1}^{n} \gamma_{pe}(G_i) + 1 = \gamma_{pe}(G_1) + \gamma_{pe}(G_2) + \cdots + \gamma_{pe}(G_n) + 1$

$$= (2 + 3 + \cdots + n + 1) + 1$$
$$= 1 + 2 + 3 + \cdots + n + n + 1$$
$$= \frac{(n+1)(n+2)}{2}$$
$$= \gamma_{pe}\left(C_{\frac{n(n+3)}{2}} \odot K_1\right).$$

Hence $\gamma_{pe}\left(C_{\frac{n(n+3)}{2}} \odot K_1\right) = \sum_{i=1}^{n} \gamma_{pe}(G_i) + 1$.

Illustration 2.4.

In Figure 2,For $n = 4$ the crown $C_{14} \odot K_1$ admits APPD into 4 −parts.



$C_{14} \odot K_1$

APDD of $C_{14}\odot K_1$

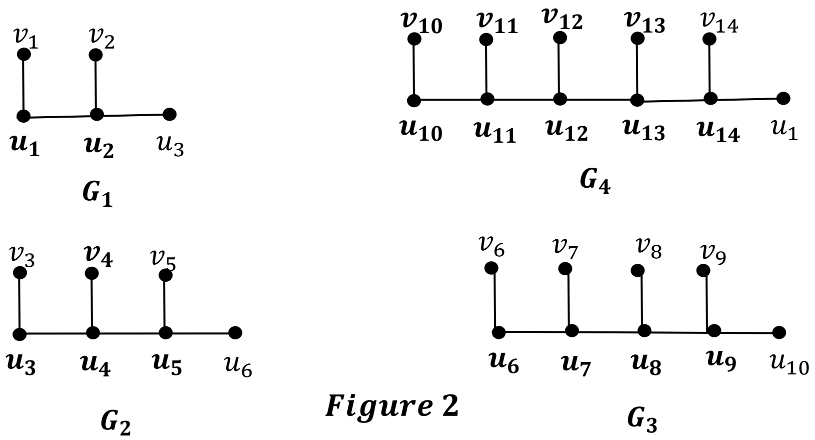


Figure 2

Here $\gamma_{pe}(G_1) = 2$, $\gamma_{pe}(G_2) = 3$, $\gamma_{pe}(G_3) = 4$ and $\gamma_{pe}(G_4) = 5$.

Theorem 2.5.

The Gear $G_{4n(n-2)}$ admits APDD into n −parts for all $n \geq 3$.

Proof .

Let $\{u, u_1, u_2, \dots, u_{4n(n-2)}\}$ be the vertex set of $G_{4n(n-2)}$.

Here u be a vertex of $G_{4n(n-2)}$ with degree $2n(n-2)$.

Define $G_1 = \langle N[u] \rangle$.

Clearly $\gamma_{pe}(G_1) = 2$.

Define $H = G_{4n(n-2)} - G_1$.

In H, $\{u_1, u_2, \dots, u_{4n(n-2)}\}$ be the vertex set of degree 2.

Define

$G_2 = P_6$

$G_3 = P_8$

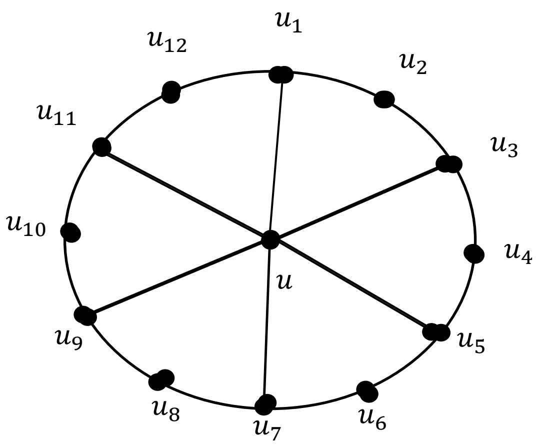
… … … … … … … … … … … … … …

$G_n = P_{2n+2}$ in H.

Clearly, $G_{4n(n-2)}$ can be decompose into $G_1, G_2, \dots, G_n$ and $\gamma_{pe}(G_i) = i + 1, 1 \leq i \leq n$. Therefore, Gear graph $G_{4n(n-2)}$ admits APDD into n −parts.

Illustration 2.6.

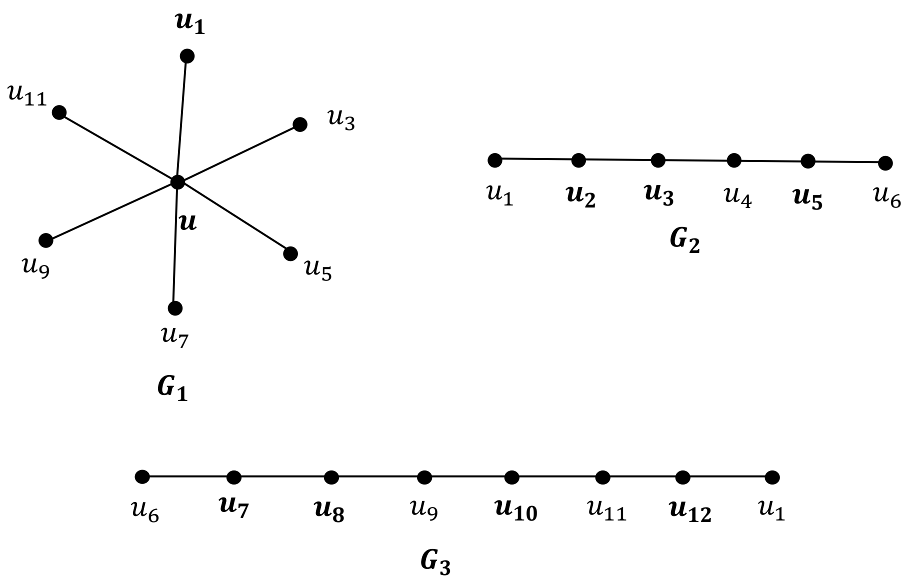
In Figure 3,For $n = 3$, G_{12} admits APDD (G_1, G_2, G_3).



$\boldsymbol{G_{12}}$

$\boldsymbol{Figure\ 3}$

$\boldsymbol{APDD\ of\ G_{12}}$



$\boldsymbol{Figure\ 3}$

Here $\gamma_{pe}(G_1) = 2$, $\gamma_{pe}(G_2) = 3$ and $\gamma_{pe}(G_3) = 4$.

Theorem 2.7.

The wheel $W_{(n+3)(n-1)}$ admits APDD into n −parts.

Proof.

Let $\{u, u_1, u_2, \dots, u_{4n(n-2)}\}$ be the vertex set of $W_{(n+3)(n-1)}$

Here u be a vertex of degree $(n+3)(n-1)$.

Define $G_1 = \langle N[u] \rangle$.

Clearly $\gamma_{pe}(G_1) = 2$.

Define $H = W_{(n+3)(n-1)} - G_1$.

Now, we decompose H as follows

Define

$G_2 = P_6$

$G_3 = P_8$

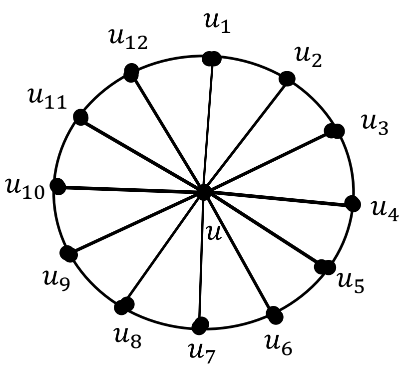
… … … … … … … … … … … … …

$G_n = P_{2n+2}$ in H.

Clearly, $W_{(n+3)(n-1)}$ can be decompose into $G_1, G_2, \dots, G_n$ and $\gamma_{pe}(G_i) = i + 1, 1 \le i \le n$. Therefore, Wheel $W_{(n+3)(n-1)}$ admits APDD into n −parts.

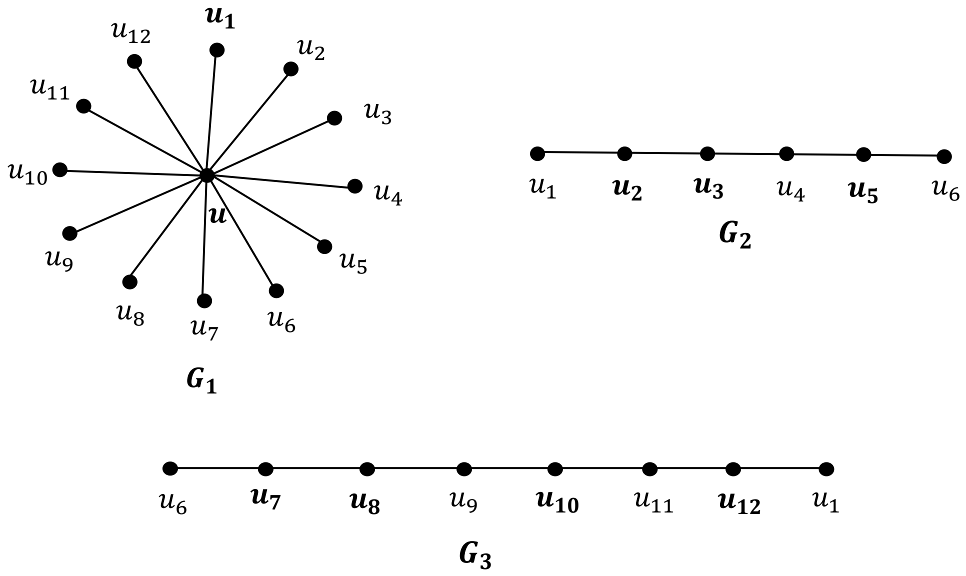
Illustration 2.8.

In Figure 4,For $n = 3$, W_{12} admits APDD into 3 −parts



$\boldsymbol{W_{12}}$

$APDD\ of\ W_{12}$



$Figure\ 4$

Here $\gamma_{pe}(G_1) = 2$, $\gamma_{pe}(G_2) = 3$ and $\gamma_{pe}(G_3) = 4$.

Theorem 2.5.

The Helm $H_{\left(\frac{(n-1)(n+4)}{2}\right)}$ admits APDD into n −parts and $\gamma_{pe}\left(H_{\frac{(n-1)(n+4)}{2}}\right) = \sum_{i=1}^{n} \gamma_{pe}(G_i) - 1$.

Proof.

Let $\{u, u_1, u_2, \dots, u_{\left(\frac{(n-1)(n+4)}{2}\right)}, v_1, v_2, \dots, v_{\left(\frac{(n-1)(n+4)}{2}\right)}\}$ be the vertex set of $H_{\left(\frac{(n-1)(n+4)}{2}\right)}$.

Here u be a vertex of $H_{\left(\frac{(n-1)(n+4)}{2}\right)}$ with degree $\frac{(n-1)(n+4)}{2}$.

Define $G_1 = \langle N(u) \rangle$.

Clearly $\gamma_{pe}(G_1) = 2$.

Define $H = H_{\left(\frac{(n-1)(n+4)}{2}\right)} - G_1$.

Here $\{u_1, u_2, \dots, u_{\left(\frac{(n-1)(n+4)}{2}\right)}\}$ be the vertex set of degree 3 in H and $\{v_1, v_2, \dots, v_{\left(\frac{(n-1)(n+4)}{2}\right)}\}$ be the pendant vertex set in H.

Define

$G_2 = \langle N[u_2, u_3] \rangle \cup (v_1, u_1)$

$G_3 = \langle N[u_5, u_6, u_7] \rangle \cup (u_4, v_4)$

… … … … … … … … … … … … …

$G_n = \langle N[u_r, u_{r+1}, \dots, u_{r+n-1}] \rangle \cup (u_{r-1}, v_{r-1})$ in H, where r can be calculated by using Newton's Divided Difference Formula.

$\boldsymbol{n}$	$\boldsymbol{r}$	$\Delta\boldsymbol{r}$	$\Delta^2\boldsymbol{r}$	$\Delta^3\boldsymbol{r}$	$\Delta^4\boldsymbol{r}$
2	2				
		3			
3	5		1		
		4		0	
4	9		1		0
		5		0	
5	14		1		
		6			
6	20				

$$n = n_0 + xh$$

$$n = 2 + x(1) \Rightarrow x = n - 2$$

$$r = r_0 + x\frac{\Delta r_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 r_0 + \cdots$$

$$= 2 + (n-2)3 + \frac{(n-2)(n-3)}{2}(1)$$

$$r = \frac{n^2 + n - 2}{2}$$

Here, $r + n - 1 = \frac{(n+4)(n-1)}{2}$.

Clearly, Helm $H_{\left(\frac{(n-1)(n+4)}{2}\right)}$ can be decompose into $G_1, G_2, \dots, G_n$ and $\gamma_{pe}(G_i) = i + 1, 1 \le i \le n$. Therefore, Helm $H_{\left(\frac{(n-1)(n+4)}{2}\right)}$ admits APDD into n −parts.

Also, the Minimal Pendant dominating set in $H_{\left(\frac{(n-1)(n+4)}{2}\right)}$ is $\{u_1, u_2, \dots, u_{\left(\frac{(n-1)(n+4)}{2}\right)}\}$ and any one vertex in the vertex set $\{v_1, v_2, \dots, v_{\left(\frac{(n-1)(n+4)}{2}\right)}\}$.

Therefore, $\gamma_{pe}\left(H_{\left(\frac{(n-1)(n+4)}{2}\right)}\right) = \frac{(n-1)(n+4)}{2} + 1 = \frac{n^2+3n-2}{2}$.

Now,$\sum_{i=1}^{n}\gamma_{pe}(G_i)-1=(2+3+\cdots+n+1)-1$

$$=1+2+3+\cdots+n+1-2$$

$$=\frac{(n+1)(n+2)}{2}-2$$

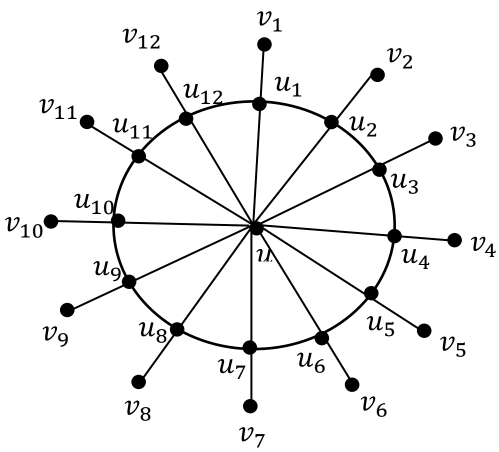
$$=\frac{n^2+3n-2}{2}$$

$$=\gamma_{pe}\left(H_{\left(\frac{(n-1)(n+4)}{2}\right)}\right).$$

Hence the theorem.

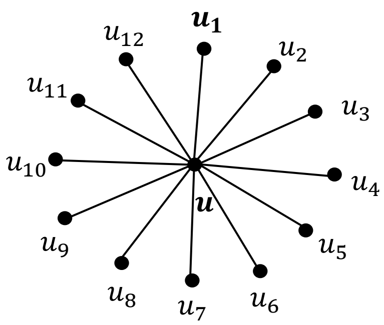
Illustration 2.10.

In Figure 5,For $n=4$,H_{12} admits APDD into 4 −parts.

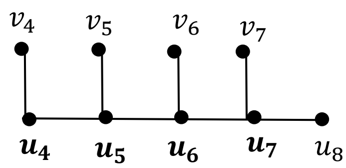


$\boldsymbol{H_{12}}$

APDD of $\boldsymbol{H_{12}}$

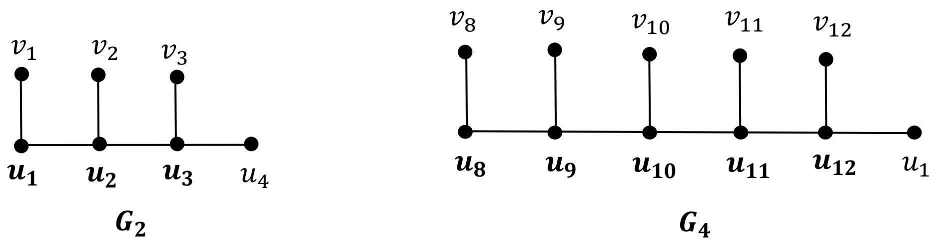


$\boldsymbol{G_1}$



$\boldsymbol{G_3}$

$\boldsymbol{Figure}$ **5**



$Figure\ 5$

Here $\gamma_{pe}(G_1) = 2$, $\gamma_{pe}(G_2) = 3$, $\gamma_{pe}(G_3) = 4$ and $\gamma_{pe}(G_4) = 5$.

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