Neutrosophic Fuzzy Ideals in Boolean like semi rings

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Abstract

The aim of this paper is to introduce the concept of Neutrosophic Fuzzy Ideals in Boolean like semi rings and also investigate some of the theorems in detail. We also obtain some characterisations and proved theorems for Boolean like semi-rings.

Keywords:

Neutrosophic fuzzy set, Neutrosophic Fuzzy Ideal and Boolean like semi ring

1. Introduction

Zadeh proposed the notion of a fuzzy set in 1965. Boolean like semi rings were introduced by K. Venkateswarlu, B.V.N. Murthy and N. Amarnath. The concept of neutrosophy was introduced by Florentin Smarandache as a new branch of philosophy. Neutrosophy is a base of Neutrosophic logic which is an extension of fuzzy logic in which indeterminancy is included. In Neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, percentage of indeterminancy in a subset I, and the percentage of falsity in a subset F.The theory of neutrosophic set have achieved great success in various fields like medical diagnosis, image processing, decision making problem ,robotics and so on. I.Arockiarani consider the neutrosophic set with value from the subset of [0,1] and extended the research in fuzzy neutrosophic set.J.Martina Jency and I.Arockia rani initiate the concept of subgroupoids in fuzzy neutrosophic set. A. Solairaju and S. Thiruveni have introduced the concept of Neutrosophic Fuzzy Ideals in near rings. R.Rajeswari and N. Meenakumari have introduced the concept of Fuzzy Bi-ideals in Boolean like semi-rings. In this paper, we recreate the concept of Neutrosophic Fuzzy Ideals in Near rings into Neutrosophic Fuzzy Ideals in Boolean like semi rings and we have discussed some of the theorems in detail.

2. Preliminaries

Definition : 2.1

A non empty set R with two binary operations '+' and '·' is called a **near-ring** if

i) (R,+) is a group

ii) (R, ·) is a semigroup

iii) $x \cdot (y+z) = x \cdot y + x \cdot z \;\; for\ all\ x, y, z \in R$

Definition : 2.2

A system (R,+, ·) a **Boolean semi ring** if and only if the following properties hold

i) (R,+) is an additive(abelian)group(whose 'zero' will be denoted by'0')

ii) (R, ·) is a semigroup of idempotents in the sense $aa = a$, for all a ∈ R

iii) $a(b+c) = ab + ac$ &

iv) $abc = bac\ for\ all\ a, b, c \in R$

Definition : 2.3

A non-empty set R together with two binary operations + and · satisfying the following conditions is called a **Boolean like semi-ring**

i) (R,+) is an abelian group

ii) (R, ·) is a semigroup

iii) $a \cdot (b+c) = a \cdot b + a \cdot c\ for\ all\ a, b, c \in R$

iv) $a + a = 0,\ for\ all\ a \in R$

v) $ab(a+b+ab) = ab, forall\ a, b \in R$

Definition : 2.4

A non-empty subset I of R is said to be an **ideal** if,

i) (I,+) is a subgroup of (R,+), (i.e) for $a, b \epsilon R \Rightarrow a + b \in R$

ii) $ra \epsilon R$ for all $a \in I, r \in R$, (i,e), $RI \subseteq I$

iii) $(r+a)s + rs \in I$ for all $r, s \in R, a \in I$.

Definition : 2.5

Let μ be a fuzzy set defined on R. Then μ is said to be a **fuzzy ideal** of R if

i) $\mu(x-y) \geq min\{\mu(x),\mu(y)\}$,for all $x,y \in R$

ii) $\mu(ra) \geq \mu(a)$,for all $r,a \in R$

iii) $\mu((r+a)s+rs) \geq \mu(a)$, for all $r,a,s \in R$

Definition: 2.6

A Neutrosophic fuzzy set A on the universe of discourse X characterized be a truth membership function $T_A(x)$,a indeterminacy function $I_A(x)$and a falsity membership function $F_A(x)$is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$ where $T_A, I_A, F_A: X \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition: 2.7

Let A and B be Neutrosophic fuzzy sets of X. Then

i) $A \cup B = \{< x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x) >: x \in X\}$,where

$T_{A\cup B}(x) = \max\{T_A(x), T_B(x)\}, I_{A\cup B}(x) = \min\{I_A(x), I_B(x)\}, F_{A\cup B}(x) = \min\{F_A(x), F_B(x)\}$,

for all $x \in X$.

ii) $A \cap B = \{< x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x) >: x \in X\}$,where

$T_{A\cap B}(x) = \min\{T_A(x), T_B(x)\}, I_{A\cap B}(x) = \max\{I_A(x), I_A(x)\}, F_{A\cap B}(x) = \max\{F_A(x), F_A(x)\}$,

for all $x \in X$.

Definition: 2.8

A Neutrosophic fuzzy set A in a near ring N is called **neutrosophic fuzzy sub near ring** of N if

i) $T_A(x-y) \geq \min\{T_A(x), T_A(y)\}, I_A(x-y) \leq \max\{I_A(x), I_A(y)\}$,

$F_A(x-y) \leq \max\{F_A(x), F_A(y)\}$

ii) $T_A(xy) \geq \min\{T_A(x), T_A(y)\}, I_A(xy) \leq \max\{I_A(x), I_A(y)\}$,

$F_A(xy) \leq \max\{F_A(x), F_A(y)\}$

3. Main Results

Definition: 3.1

Let R be a Boolean like semi ring. A Neutrosophic fuzzy set A in a Boolean like semi ring R is called Neutrosophic fuzzy ideal of R if

(i) $T_A(x-y) \geq \min\{T_A(x), T_A(y)\}, I_A(x-y) \leq \max\{I_A(x), I_A(y)\}$,
$F_A(x-y) \leq \max\{F_A(x), F_A(y)\}$

(ii) $T_A(ra) \geq T_A(a), I_A(ra) \leq I_A(a), F_A(ra) \leq F_A(a)$

(iii) $T_A((r+a)s+rs) \geq T_A(a), I_A((r+a)s+rs) \leq I_A(a)$,
$F_A\big((r+a)s+rs)\big) \leq F_A(a)$

Theorem: 3.2

Let A and B be neutrosophic fuzzy ideals of R. If $A \subset B$, then $A \cup B$ is a neutrosophic fuzzy ideal of R.

Proof:

Let A and B are Neutrosophic fuzzy ideals of R. Let $x, y, z \in R$

i)
$$\begin{aligned} T_{A\cup B}(x-y) &= \max\{T_A(x-y), T_B(x-y)\} \\ &\geq \max\{\min\{T_A(x), T_A(y)\}, \min\{T_B(x), T_B(y)\}\} \\ &= \min\{\max\{T_A(x), T_B(x)\}, \max\{T_A(y), T_B(y)\}\} \\ &= \min\{T_{A\cup B}(x), T_{A\cup B}(y)\} \end{aligned}$$

$$\begin{aligned} I_{A\cup B}(x-y) &= \min\{I_A(x-y), I_B(x-y)\} \\ &\leq min\{\max\{I_A(x), I_A(y)\}, \max\{I_B(x), I_B(y)\}\} \\ &= \max\{\min\{I_A(x), I_B(x)\}, \min\{I_A(y), I_B(y)\}\} \\ &= \max\{I_{A\cup B}(x), I_{A\cup B}(y)\} \end{aligned}$$

$$\begin{aligned} F_{A\cup B}(x-y) &= \min\{F_A(x-y), F_B(x-y)\} \\ &\leq min\{\max\{F_A(x), F_A(y)\}, \max\{F_B(x), F_B(y)\}\} \\ &= \max\{\min\{F_A(x), F_B(x)\}, \min\{F_A(y), F_B(y)\}\} \\ &= \max\{F_{A\cup B}(x), F_{A\cup B}(y)\} \end{aligned}$$

ii)
$$\begin{aligned} T_{A\cup B}(ra) &= \max\{T_A(ra), T_B(ra)\} \\ &\geq \max\{T_A(a), T_B(a)\} \end{aligned}$$

$$= T_{A\cup B}(a)$$

$$I_{A\cup B}(ra) = \min\{I_A(ra), I_B(ra)\}$$

$$\leq \min\{I_A(a), I_B(a)\}$$

$$= I_{A\cup B}(a)$$

$$F_{A\cup B}(ra) = \min\{F_A(ra), F_B(ra)\}$$

$$\leq \min\{F_A(a), F_B(a)\}$$

$$= F_{A\cup B}(a)$$

iii) $T_{A\cup B}\big((r+a)s + rs\big) = \max\big\{T_A\big((r+a)s+rs\big), T_B\big((r+a)s+rs\big)\big\}$

$$\geq \max\{T_A(a), T_B(a)\} = T_{A\cup B}(a)$$

$$I_{A\cup B}\big((r+a)s + rs\big) = \min\big\{I_A\big((r+a)s+rs\big), I_B\big((r+a)s+rs\big)\big\}$$

$$\leq \min\{I_A(a), I_B(a)\} = I_{A\cup B}(a)$$

$$F_{A\cup B}\big((r+a)s + rs\big) = \min\big\{F_A\big((r+a)s+rs\big), F_B\big((r+a)s+rs\big)\big\}$$

$$\leq \min\{F_A(a), F_B(a)\} = F_{A\cup B}(a)$$

Therefore, $A \cup B$ is a Neutrosophic fuzzy ideal of R.

Theorem: 3.3

Let A and B be neutrosophic fuzzy ideals of R. Then $A\cap B$ is a neutrosophic fuzzy ideal of R.

Proof:

Let A and B are Neutrosophic fuzzy ideals of R. Let $x, y, z \in R$

i) $T_{A\cap B}(x-y) = \min\{T_A(x-y), T_B(x-y)\}$

$$\geq \min\{\min\{T_A(x), T_A(y)\}, \min\{T_B(x), T_B(y)\}\}$$

$$= \min\{\min\{T_A(x), T_B(x)\}, \min\{T_A(y), T_B(y)\}\}$$

$$= \min\{T_{A\cap B}(x), T_{A\cap B}(y)\}$$

$$I_{A\cap B}(x-y) = \max\{I_A(x-y), I_B(x-y)\}$$

$$\leq max\{\max\{I_A(x), I_A(y)\}, \max\{I_B(x), I_B(y)\}\}$$

$$= \max\{\max\{I_A(x), I_B(x)\}, \max\{I_A(y), I_B(y)\}\}$$

$$= \max\{I_{A\cap B}(x), I_{A\cap B}(y)\}$$

$$F_{A\cap B}(x-y) = \max\{F_A(x-y), F_B(x-y)\}$$
$$\leq max\{\max\{F_A(x), F_A(y)\}, \max\{F_B(x), F_B(y)\}\}$$
$$= \max\{\max\{F_A(x), F_B(x)\}, \max\{F_A(y), F_B(y)\}\}$$
$$= \max\{F_{A\cap B}(x), F_{A\cap B}(y)\}$$

ii) $T_{A\cap B}(ra) = \min\{T_A(ra), T_B(ra)\}$
$$\geq \min\{T_A(a), T_B(a)\}$$
$$= T_{A\cap B}(a)$$

$$I_{A\cap B}(ra) = \max\{I_A(ra), I_B(ra)\}$$
$$\leq \max\{I_A(a), I_B(a)\}$$
$$= I_{A\cap B}(a)$$

$$F_{A\cap B}(ra) = \max\{F_A(ra), F_B(ra)\}$$
$$\leq \max\{F_A(a), F_B(a)\}$$
$$= F_{A\cap B}(a)$$

iv) $T_{A\cap B}\big((r+a)s+rs\big) = \min\big\{T_A\big((r+a)s+rs\big), T_B\big((r+a)s+rs\big)\big\}$
$$\geq \min\{T_A(a), T_B(a)\} = T_{A\cap B}(a)$$

$$I_{A\cap B}\big((r+a)s+rs\big) = \max\big\{I_A\big((r+a)s+rs\big), I_B\big((r+a)s+rs\big)\big\}$$
$$\leq \max\{I_A(a), I_B(a)\} = I_{A\cap B}(a)$$

$$F_{A\cap B}\big((r+a)s+rs\big) = \max\big\{F_A\big((r+a)s+rs\big), F_B\big((r+a)s+rs\big)\big\}$$
$$\leq \max\{F_A(a), F_B(a)\} = F_{A\cap B}(a)$$

Therefore, $A \cap B$ is a Neutrosophic fuzzy ideal of R.

Lemma: 3.4

For all $a, b \in I$ and i is any positive integer, if $a = b$,then

i) $a^i \leq b^i$

ii) $[\min(a,b)]^i = \min(a^i, b^i)$

iii) $[\max(a,b)]^i = \max(a^i, b^i)$

Theorem: 3.5

Let A be a Neutrosophic fuzzy ideal of R. Then $A^m = \{< x, T_{A^m}(x), I_{A^m}(x), F_{A^m}(x) > : x \in R\}$ is a Neutrosophic fuzzy ideal of R^m, where m is a positive integer and $T_{A^m}(x) = (T_A(x))^m, I_{A^m}(x) = (I_A(x))^m, F_{A^m}(x) = (F_A(x))^m$.

Proof:

Let A be a Neutrosophic fuzzy ideal of R. Let $x, y, z \in R$.

i) $T_{A^m}(x-y) = (T_A(x-y))^m$

$\geq [\min\{T_A(x), T_A(y)\}]^m$

$= min\{T_{A^m}(x), T_{A^m}(y)\}$

$I_{A^m}(x-y) = (I_A(x-y))^m$

$\leq [\max\{I_A(x), I_A(y)\}]^m$

$= max\{I_{A^m}(x), I_{A^m}(y)\}$

$F_{A^m}(x-y) = (F_A(x-y))^m$

$\leq [\max\{F_A(x), F_A(y)\}]^m$

$= \max\{F_{A^m}(x), F_{A^m}(y)\}$

ii) $T_{A^m}(ra) = (T_A(ra))^m$

$\geq \left(T_A(a)\right)^m$

$= T_{A^m}(a)$

$I_{A^m}(ra) = (I_A(ra))^m$

$\leq \left(I_A(a)\right)^m$

$= I_{A^m}(a)$

$F_{A^m}(ra) = (F_A(ra))^m$

$\leq \left(F_A(a)\right)^m$

$= F_{A^m}(a)$

iii) $T_{A^m}((r+a)s+rs) = (T_A\left((r+a)s+rs\right))^m$

$\geq \left(T_A(a)\right)^m$

$= T_{A^m}(a)$

$I_{A^m}\left((r+a)s+rs\right) = (I_A\left((r+a)s+rs\right))^m$

$$\leq \left(I_A(a)\right)^m$$
$$= I_{A^m}(a)$$
$$F_{A^m}\big((r+a)s+rs\big) = (F_A\big((r+a)s+rs\big))^m$$
$$\leq \left(F_A(a)\right)^m$$
$$= F_{A^m}(a)$$

Therefore, A^m is a Neutrosophic fuzzy ideal of R^m.

4. References

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