Fuzzy Soft Ternary Γ-Semirings-IV

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ABSTRACT

In this paper we define fuzzy soft prime t-ideal(FSP tideal), completely prime t-ideal(CP tideal), fuzzy soft completely prime t-ideal(FSCP tideal), zero symmetric ternary Γ-semiring and we proved that if (f, F_1, Γ) is a FSP tideal over F then $f_x(p)$ *for all* $x \in F_1$ and $f_x(p)$ taking exactly 2 values and a Fuzzy Soft(FS) Set (f, F_1, Γ) over F is a FSP tideal over F iff for each f_x *where* $x \in F_1, (f_x)_t, t \in \operatorname{Im} f_x$ is a prime tideal of F. and also for each $x \in F_1, N = \{p \in F / f_x(p) = f_x(0)\}$ is a CP tideal of F if (f, F_1, Γ) is a FSCP tideal over F.

Keywords: Fuzzy soft ternary Γ- Semiring (FSTΓ-SR), Fuzzy soft tideal, Fuzzy soft prime tideal, Fuzzy soft completely prime tideal.

1. INTRODUCTION

The fuzzy set concept was introduced by Zadeh [18] in 1965. In the year 2001 SS theory extended to FSS theory by Maji et.al.[9]. The authors in their earlier papers have introduced FSTΓ-SR, FS tideal, FS quasi tideals, FS bi-ideals and FS interior tideals & their basic algebraic properties in FSTΓ-SR. In this paper the authors are trying to define fuzzy soft prime tideals.

2. PRELIMINARIES

For preliminaries of fuzzy soft ternary Γ- Semiring refer to references [19], [20]..

3. MAIN RESULTS

Fuzzy soft prime tideals:

Def 3.1: Let F be a FSTΓ-SR and F_2, F_3, F_4 be subsets of $F_1 \subseteq E$(E is a parameter set of F), then a FS tideal (f, F_1, Γ) is a FSP tideal over F if it is not a constant & if $\exists$ non-absolute FS tideals $(g, F_2, \Gamma), (h, F_3, \Gamma), (i, F_4, \Gamma)$ over F such that $(g, F_2, \Gamma) o (h, F_3, \Gamma) o (i, F_4, \Gamma) \subseteq (f, F_1, \Gamma)$ $\Rightarrow either\,(g, F_2, \Gamma) \subseteq (f, F_1, \Gamma)\, or\; (h, F_3, \Gamma) \subseteq (f, F_1, \Gamma)\, or\, (i, F_4, \Gamma) \subseteq (f, F_1, \Gamma).$

Ex 3.2: Let F= {0, p, q, r}, Γ= {o, r}. and define [ ] on F by $[p\alpha q\beta r] = (p\alpha q)\beta r$ for every $p, q, r \in F\; and\; \alpha, \beta \in \Gamma$ where '+' and '.' are defined as follows

+	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p
r	r	q	p	0

Table 1

.	0	p	q	r
0	0	0	0	0
p	0	0	0	0
q	0	0	0	0
r	0	p	q	r

Table 2

Then $(F, +, [\ ], \Gamma)$ is a FSTΓ-SR and {0, p} & {0, q} are tideals of F.

Now consider the tideals P= {0}, Q= {0, p}, R= {0, q} in F, then $[P\Gamma Q\Gamma R] \subseteq \{0\}$ $and\; P \subseteq \{0\}$ and then {0} is a prime tideal of F. Again if E=F. Let F_1=E, F_2= {0}, F_3= {0, p}, F_4= {0, q}. Let (f, F_1, Γ), $(g, F_2, \Gamma), (h, F_3, \Gamma)\, and\, (j, F_4, \Gamma)$ defined as

$$f_x(p) = \begin{cases} 1 & , if\; p = 0 \\ 0.9, & if\; p \neq 0 \end{cases} \quad for\; every\; x \in F_1, \quad g_y(p) = \begin{cases} 1 & , if\; p = 0 \\ 0.1, & if\; p \neq 0 \end{cases},$$

$$h_z(s) = \begin{cases} 1 & , if\; s = 0 \\ 0.7, & if\; s = q, r \end{cases} \quad r_w(s) = \begin{cases} 1 & , if\; s = 0, q \\ 0.8, & if\; s = p, r \end{cases} \quad for\; all\; y \in F_2, z \in F_3, w \in F_4$$

then $(g, F_2, \Gamma), (h, F_3, \Gamma), (j, F_4, \Gamma)$ are all FS tideals over F.

Obviously if $(g, F_2, \Gamma) o (h, F_3, \Gamma) o (j, F_4, \Gamma) = (k, F_5, \Gamma)$ where

$F_5 = F_2 \cap F_3 \cap F_4 = \{0\}$,$then\; for\; l \in F_5$

$$k_l(v) = \begin{cases} 1 & , if\; v = [p\alpha q\beta r] \\ 0, & if\;\; otherwise \end{cases}.$$

Hence $(g, F_2, \Gamma) o (h, F_3, \Gamma) o (j, F_4, \Gamma) \subseteq (f, F_1, \Gamma)$ and $(g, F_2, \Gamma) \subseteq (f, F_1, \Gamma)$ .Thus (f, F_1, Γ) is a FSP tideal of F.

Def 3.3: An tideal H of an FSTΓ-SR F is called a CP tideal of F if for $p, q, r \in F$ $and\; \alpha, \beta \in \Gamma$, $[p\alpha q\beta r] \in H \Rightarrow either\; p \in H\; or\; q \in H\; or\; r \in H$

Ex 3.4: Let F={0,p,q,r}, Γ={0,r} and [ ] be defined by $[p\alpha q\beta r]=(p\alpha q)\beta r$ for $p,q,r\in F,\alpha,\beta\in\Gamma.$ where + and . are defined as follows

+	0	p	q	r
0	0	p	q	r
p	p	0	**r**	q
q	q	r	0	p
r	r	q	p	0

Table 3

.	0	p	q	r
0	0	0	0	0
p	0	0	0	0
q	0	0	q	q
r	0	0	q	q

Table 4

then {0,p} is a CP tideal of F.

Def 3.5: A FS tideal (f,F_1,Γ) over F is called a FSCP tideal if for all $p,q,r\in F,\alpha,\beta\in\Gamma$, $f_x([p\alpha q\beta r])\le\max\{f_x(p),f_x(q),f_x(r)\}.$

Ex 3.6: As in Ex 3.4 Let E=F, let F_1= {0, 3} Define (f,F_1,Γ) as follows. For every $p\in F$

$f_x(p)=\begin{cases}1 & ,if\ p=0,3\\ 0.8, & if\ p\neq 0,3\end{cases}$ $for\ every\ x\in F_1,$ then (f,F_1,Γ) is a FSCP tideal over F.

Def 3.7: Let F be a ternary Γ-Semiring then $N_0=\{n\in[no0]=0\}$ is zero symmetric (ZS) of F. If F=N_0 then F is called a ZS ternary Γ-Semiring.

Th 3.8: If (f,F_1,Γ) is a FSS over F and if $x\in F_1$, $N=F_{f_{F_1}}^{f_{F_1}(0)}=\{p\in F/f_x(p)=f_x(0)\}$ then N is a prime tideal of F if (f,F_1,Γ) is a FSP tideal over F.

Proof: Let $F_1\subseteq E$ (Parameter set of F). Since (f,F_1,Γ) is a FS tideal over F, N is a tideal of F. Let P, Q, R be tideals of F such that $[P\Gamma Q\Gamma R]\subseteq N$ --(1)

Let $(g,F_2,\Gamma),(h,F_3,\Gamma),(j,F_4,\Gamma)$ be FSSs over F. Where F_2, F_3, F_4 are subsets of F_1 for $p,q,r\in F$

Define $g_y(p)=\begin{cases}f_x(0) & ,if\ p\in P\\ 0\ , & otherwise\end{cases}$, $h_z(q)=\begin{cases}f_x(0) & ,if\ q\in Q\\ 0, & otherwise\end{cases}$,

$j_w(r)=\begin{cases}f_x(0) & ,if\ r\in R\\ 0, & otherwise\end{cases}$, $for\ all\ y\in F_2,z\in F_3,w\in F_4$

then by Th 3.18 [19] $(g,F_2,\Gamma),(h,F_3,\Gamma),(j,F_4,\Gamma)$ are FS tideals over F. Now we have to prove that $(g,F_2,\Gamma)o(h,F_3,\Gamma)o(j,F_4,\Gamma)\subseteq(f,F_1,\Gamma).$

i.e, it is enough to show that $(g_y o h_z o j_w)(v)\le f_x(v)\ for\ all\ v\in F.$ obviously $F_2\cap F_3\cap F_4\subseteq F_1.$ If $v\in F$ then we have $i)\ v\in N\ ii)\ v\notin N$

(i) Let $v\in N$ then $v=0\ or\ v\neq 0.$ If v=0 then as F is zero symmetric $(g_y o h_z o j_w)(v)=f_x(0)$

$\Rightarrow (g_y o h_z o j_w)(v) = f_x(v)\ for\ all\ v \in F.$ If $v \neq 0$ then $(g_y o h_z o j_w)(v) = f_x(0)\ or\ 0$ depending on v is expressed as $[p\alpha q\beta r]\ or\ not$. In either case as $v \in N$, $f_x(v) = f_x(0) > 0.$ Hence $(g_y o h_z o j_w)(v) \leq f_x(v).$

(ii) Suppose $v \notin N\ then\ f_x(v) \neq f_x(0).$ since $f_x(0) > 0\ and\ f_x(v) \neq f_x(0).$ we have $f_x(v) = 0\ or\ f_x(0) > 0.$ Also since $v \notin N\ if\ v = [p\alpha q\beta r]$ then by (i) either p, q, r are not in P, Q, R respectively or at least one of p, q, r is in not respectively in P, Q, R.

Thus $(g_y o h_z o j_w)(v) = \sup\{g_y(p) \wedge h_z(q) \wedge j_w(r)\} = 0 \leq f_x(v).$ So from the above we have $(g_y o h_z o j_w)(v) \leq f_x(v)$ for all $v \in F$. Hence $(g, F_2, \Gamma) o (h, F_3, \Gamma) o (j, F_4, \Gamma)$ $\subseteq (f, F_1, \Gamma)$. Since (f, F_1, Γ) is a FSP tideal over F either $(g, F_2, \Gamma) \subseteq (f, F_1, \Gamma)\ or$ $(h, F_3, \Gamma) \subseteq (f, F_1, \Gamma)\ or\ (i, F_4, \Gamma) \subseteq (f, F_1, \Gamma).$

Let $(g, F_2, \Gamma) \subseteq (f, F_1, \Gamma)$ and P is not a subset of N. Then there exits $p \in P\ and\ p \notin N \Rightarrow f_x(p) \neq f_x(0).$ But $f_x(0) \geq f_x(p).$ [$Since\ f_x(0) = f_x(p-p)$ $\geq \min\{f_x(p), f_x(p-p)\}$] Thus $f_x(p) \leq f_x(0).$ Now $g_y(p) = f_x(0) > f_x(p).$ which contradicts the assumption $(g, F_2, \Gamma) \subseteq (f, F_1, \Gamma). \Rightarrow P \subseteq N,$ Similarly $(h, F_3, \Gamma) \subseteq (f, F_1, \Gamma)\ or\ (i, F_4, \Gamma) \subseteq (f, F_1, \Gamma) \Rightarrow F_2 \subseteq T\ or\ F_3 \subseteq T$. Therefore N is a Prime tideal.

Corollary3.9: Let (f, F_1, Γ) be a FSS over F and $N = {F^1}_{f_x} = \{p \in F / f_x(p) = 1\}$ then N is a prime tideal of F if (f, F_1, Γ) is a FSP tideal over F

Proof: In the above theorem take $f_x(0) = 1\ in\ {F_{f_x}}^{f_x(0)}$

Th 3.10: If (f, F_1, Γ) is a FSP tideal over F then $f_x(0) = 1\ \ for\ all\ x \in F_1$

Proof: Let $f_x(0) < 1$. Since (f, F_1, Γ) is not a constant, $\exists\ p \in F_1\ such\ that\ f_x(p) < f_x(0)$

Define $(g, F_2, \Gamma), (h, F_3, \Gamma), (j, F_4, \Gamma)$ where F_2, F_3, F_4 are subsets of F_1.

$for\ p, q, r \in F, \alpha, \beta \in \Gamma$. $g_y(p) = \begin{cases} 1, if\ \ f_x(p) = f_x(0) \\ 0, \quad otherwise \end{cases}$ $h_z(q) = f_x(0)$

$j_w(r) = \begin{cases} 1, if\ \ f_x(r) = f_x(0) \\ 0, \quad otherwise \end{cases}$ $for\ each\ y \in F_2, z \in F_3, w \in F_4$ then

$(g, F_2, \Gamma), (h, F_3, \Gamma)\ and\ (j, F_4, \Gamma)$ are all FS tideals over F.

Since $(g_y)(0) = 1 > f_x(0) > f_x(p), (h_z)(0) = f_x(0) > f_x(p),$ $(j_w)(0) = 1 > f_x(0) > f_x(p), (g, F_2, \Gamma), (h, F_3, \Gamma)\ and\ (j, F_4, \Gamma)$ are not subsets of (f, F_1, Γ)---(1)

Now we prove that $(g, F_2, \Gamma) o (h, F_3, \Gamma) o (j, F_4, \Gamma) \subseteq (f, F_1, \Gamma)$ (i.e) to prove $(g_y o h_z o j_w)(v) \leq f_x(v)\ for\ every\ v \in F$. obviously $F_2 \cap F_3 \cap F_4 \subseteq F_1$. If $v \in F$ then we have $(i)\ v = 0\ (ii)\ v \neq 0$

$(i)\, If\ v=0 \Rightarrow f_x(v)=f_x(0)$

$(g_y o h_z o j_w)(v)=\sup\{f_x(0),0\}$ then $(g_y o h_z o j_w)(v)=f_x(0)=f_x(v)\ \ for\ every\ v\in F$

(ii) Let $v\neq 0\ and\ v\neq[p\alpha q\beta r]$ then $(g_y o h_z o j_w)(v)=0\leq f_x(v)$. If $v=[p\alpha q\beta r]$, $(g_y o h_z o j_w)(v)\neq 0$ and if $g_y(p)\, or\, h_z(q)$ is zero, then $g_y(p)\wedge h_z(q)\wedge j_w(r)=0$ and hence $(g_y o h_z o j_w)(v)=0\leq f_x(v)$.

Also if $g_y(p)\, and\ j_w(p)$ are non-zero $g_y(p)\wedge h_z(q)\wedge j_w(r)=f_x(0)=f_x(p)\leq f_x(v)$ by the definitions of $(g,F_2,\Gamma),(h,F_3,\Gamma)\, and\,(j,F_4,\Gamma)$ is a FS tideal. Hence from the above $(g_y o h_z o j_w)(v)\leq f_x(v)\ for\ all\ v\in F$. Thus $(g,F_2,\Gamma)o(h,F_3,\Gamma)o(j,F_4,\Gamma)\subseteq(f,F_1,\Gamma)$ Since (f,F_1,Γ) is a FSP tideal then $(g,F_2,\Gamma)\subseteq(f,F_1,\Gamma)$ or $(h,F_3,\Gamma)\subseteq(f,F_1,\Gamma)$ or $(j,F_4,\Gamma)\subseteq(f,F_1,\Gamma)$. which contradicts (i). So $f_x(0)=1$.

If the range of f_x is singleton then (f,F_1,Γ) be a constant tideal. Hence if (f,F_1,Γ) is a non-constant FS tideal then f_x takes more than one value of F.

Th 3.11: Let $H(\neq\phi)$ be subset of a FSTΓ-SR F and Π_H be the characteristic function of H, then H is a tideal of F iff Π_H is a tideal of F. Where $(\Pi_H)_e(x)=\begin{cases}1, & if\ x\in H\\ 0, & if\ x\notin H\end{cases}\ for\ all\ e\in E.$

Proof: Let $H(\neq\phi)$ be a tideal of a $FST\Gamma-SR$ F. Let $p,q,r\in F,\alpha,\beta\in\Gamma$, then $p+q\in H$, $p\alpha q\beta r\in H$ if $r\in H$, it follows that $(\Pi_H)_e(p+q)=1$ and $(\Pi_H)_e(p\alpha q\beta r)=1=(\Pi_H)(r)$. If $q\notin H,\ then(\Pi_H)_e(r)=0$. In this case $(\Pi_H)_e(p\alpha q\beta r)\geq 0=(\Pi_H)_e(r)$. Therefore $(\Pi_H)_e$ is a FS left tideal of F. Similarly we can prove that $(\Pi_H)_e$ is a lateral as well as right FS tideal of F. So $(\Pi_H)_e$ is a FS tideal of F.

Conversely, suppose that $(\Pi_H)_e$ be a FS tideal of F. Let $p,q\in H,\, then\, if\ p,q\in H$ $\Rightarrow(\Pi_H)_e(p)=(\Pi_H)_e(q)=1$ and $(\Pi_H)_e(p+q)\geq\min\{(\Pi_H)_e(p),(\Pi_H)_e(q)\}$ $=\min\{1,1\}=1$. Thus $p+q\in H,(\Pi_H)_e(p)=(\Pi_H)_e(q)=1$. Now let $p\in H,s,t\in F$, $\alpha,\beta\in\Gamma$, then $(\Pi_H)_e(p)=1$ also $(\Pi_H)_e(s\alpha t\beta p)\geq(\Pi_H)_e(p)=1$. Thus $s\alpha t\beta p\in H$.
Thus H is a left tideal of F.
In the same way, we prove that H is a lateral, right tideal of F. ⇒ H is a tideal of F.

Th3.12: If (f,F_1,Γ) is a FSP tideal over F then for x ∈ F_1, $f_x(p)$ taking exactly 2 values.

Proof: Let (f,F_1,Γ) be a FSP tideal over F. $Then\ f_x(0)=1$. To prove, $f_x(p)$ takes only 2 values. Suppose $f_x(p)$ takes more than two values. Then ∃ p, q ∈ F such that $f_x(p)=s,\ f_x(q)=t$. For the sake of definiteness, let $0 < s < t < 1$. Now, let $P=\{p\in F/f_x(p)\geq s\},R=\{r\in F/f_x(r)\geq t\}$ Then P and R are tideals of F Th 3.18 [19].Let F_2, F_3, be subsets of F_1 and define $(g,F_2,\Gamma),(h,F_3,\Gamma)\ \ and\,(j,F_4,\Gamma)$ over F as follows

$g_y(p)=\begin{cases}1, if\ p\in P\\ 0,\ otherwise\end{cases}$ $h_z(p)=\frac{s+t}{2}=l(say)$ $j_w(r)=\begin{cases}1, if\ r\in R\\ 0,\ otherwise\end{cases}$

$for\ every\ y\in F_2, z\in F_3, w\in F_4$.Then by Th 3.11 $(g,F_2,\Gamma),(h,F_3,\Gamma)\ and\ (j,F_4,\Gamma)$ are FS tideals over F. Hence (h,F_3,Γ) is a FS tideal over F. Since $r>0,(g,F_2,\Gamma)\neq(1,E)$

To establish that $(g,F_2,\Gamma)o(h,F_3,\Gamma)o(j,F_4,\Gamma)\subseteq(f,F_1,\Gamma)$ i.e $(g_y o h_z o j_w)(v)\leq f_x(v)$ $for\ all\ v\in F$. Since $u\in F$ then we have $(i)\ v=0\ (ii)\ v\neq 0$

(i) $If\ v=0\Rightarrow f_x(0)=f_x(v)=1$ Since $r<1=f_x(0), f_x(0)>r\ and\ hence\ 0\in P$ and therefore, $g_y(0)=1$. Obviously $h_z(0)=1$. Since $t<1=f_x(0), f_x(0)>t\ and\ hence\ 0\in R$ and therefore $j_w(0)=1$. . Thus if $v=0,(g_y o h_z o j_w)(v)=1\ or\ l$. Since $l<1=f_x(0)=f_x(v)$, $(g_y o h_z o j_w)(v)\leq f_x(v)$.

(ii) Let *v* be any non-zero element of F such that $f_x(v)\geq t$ .Then $v\in Z\Rightarrow j_w(v)=1\Rightarrow$. Obviously $h_z(v)=l$. Since $f_x(v)\geq t>r, g_y(v)=1$. Thus $g_y(p)\wedge h_z(q)\wedge j_w(r)=1\wedge l\wedge 1=l$. $\Rightarrow(g_y o h_z o j_w)(v)=l<t\leq f_x(v)$. Now, let v be any non-zero element of F such that $f_x(v)<t. Then\ j_w(v)=0$. Then $\Rightarrow(g_y o h_z o j_w)(v)=0\leq f_x(v)$. Thus, in both the cases $(g_y o h_z o j_w)(v)\leq f_x(v),\forall v\in F$. Hence $(g,F_2,\Gamma)o(h,F_3,\Gamma)o(j,F_4,\Gamma)\subseteq(f,F_1,\Gamma)$.

Now we have (f,F_1,Γ) is a FS Prime tideal, either $(g,F_2,\Gamma)\subseteq(f,F_1,\Gamma)\ or$ $(h,F_3,\Gamma)\subseteq(f,F_1,\Gamma)\ or\ (j,F_4,\Gamma)\subseteq(f,F_1,\Gamma)$ ----(1)

Since $f_x(p)=r, g_y(p)=1>r\Rightarrow g_y(p)>f_x(p)$. Implies $(g,F_2,\Gamma)\not\subseteq(f,F_1,\Gamma)$ Similarly $(h,F_3,\Gamma)\not\subseteq(f,F_1,\Gamma)$, $(j,F_4,\Gamma)\not\subseteq(f,F_1,\Gamma)$ Which contradicts (1).Hence $f_x(p)$ cannot take more two values in [0, 1].

Th 3.13: (f,F_1,Γ) be a FS set over F and $f_x(0)=1$. If $F^1{}_{f_x}=\{p\in F / f_a(p)=1\}$ is a tideal of F, then (f,F_1,Γ) be a FS tideal over F.

Proof: Let $r,s\in F^1{}_{f_x}$ then $f_x(r)=1\ and\ f_x(s)=1$. As (f,F_1,Γ) is a FSTΓ-SR and $r,s\in F,\ f_x(r+s)\geq\min\{f_x(r),f_x(s)\}=1$. But as we know that $1\geq f_x(r+s)$. Hence $f_x(r+s)=1$ which implies that $r+s\in F^1{}_{f_x}$. If $F^1{}_{f_x}$ is a FS tideal over F, for each $x\in F_1,\alpha,\beta\in\Gamma, r\in F$. Let $1=f_x(r)\ then\ r\in F^1{}_{f_x}$. Since $F^1{}_{f_x}$ is a tideal over F, then $s\alpha r\beta t\in F^1{}_{f_x}$ for each $s,t\in F$. This means that $f_x(s\alpha r\beta t)\geq f_x(r)=1$ i.e f_x is a FS lateral tideal of F. Similarly we can prove that f_x is a FS left as well as right tideal, and so (f,F_1,Γ) is a FS tideal over F.

Th 3.14: Let (f,F_1,Γ) be a FS set over F and for x ∈ F_1, $|Im\ f_x|=2$ and $f_x(0)=1$ and $N=\{p\in F / f_x(p)=1\}$ is a prime tideal of F then (f,F_1,Γ) is a FSP tideal over F

Proof: By Th 3.13, if N is a tideal of F then (f,F_1,Γ) is a FS tideal over F.

Let $(g,F_2,\Gamma),(h,F_3,\Gamma)\, and\, (j,F_4,\Gamma)$ be FS tideals over F where $F_2,F_3,F_4 \subseteq F_1 \subseteq E$ with $(g,F_2,\Gamma)o(h,F_3,\Gamma)o(j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)-(1)$. suppose $(g,F_2,\Gamma) \not\subseteq (f,F_1,\Gamma)$, $(h,F_3,\Gamma) \not\subseteq (f,F_1,\Gamma)$, $(j,F_4,\Gamma) \not\subseteq (f,F_1,\Gamma)$. Then $g_y(p) > f_x(p), h_z(q) > f_x(q)$, $j_w(r) > f_x(r)-(2)$ for some $p,q,r \in F$. But $f_x(p)=1=f_x(0)\,\forall p \in F$. From (2) $p,q,r \notin N$.($\because$ if p, q, r ∈ N then $f_x(p)=1=f_x(q)=f_x(r)$ which from (2) will imply that $g_y(p)>1, h_z(q)>1, j_w(r)>1$ which is impossible).Since $p,q,r \notin N, \alpha,\beta \in \Gamma$, $[p\alpha q\beta r] \notin N \;\Rightarrow f_a([p\alpha q\beta r]) \neq 1$. Now, consider,

$$(g_y o h_z o j_w)(v) = \bigvee_{v=[p\alpha q\beta r]} g_y(p) \wedge h_z(q) \wedge j_w(r)\; from (1)$$

$$\begin{aligned} f_x(v) = f_x([p\alpha q\beta r]) &\geq \bigvee_{v=[p\alpha q\beta r]} g_y(p) \wedge h_z(q) \wedge j_w(r) \\ &\geq g_y(p) \wedge h_z(q) \wedge j_w(r) \\ &\succ f_x(p) \wedge f_x(q) \wedge f_x(r) \qquad ---(3) \end{aligned}$$

Since $|\mathrm{Im}\, f_x| = 2$ and $f_x(p) \neq 1, f_x(p) = s$. Similarly $f_x(q) = s = f_x(t)$ .Also $f_x(v) \neq 1$ and hence $f_x(v) = s$. Thus from (3) we get $s > s$, a contradiction. So $(g,F_2,\Gamma) \subseteq (f,F_1,\Gamma)$ or $(h,F_3,\Gamma) \subseteq (f,F_1,\Gamma)$ or $(j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$. Therefore (f,F_1,Γ) is a FSP tideal over F.

Th 3.15: (i) A subset $H(\neq \phi)$ of F is a prime tideal iff (f,F_1,Γ) is a FSP tideal over F where $f_x(p) = \begin{cases} 1, if\ p \in H \\ t, if\ p \notin H \end{cases}$, $0 \leq t < 1$ for every $x \in F_1$.

(ii) In particular an tideal I of F is a prime tideal iff (ψ_I, D) be a FSP tideal over F where $\psi_I : D \to I^F$ is defined by $(\psi_I)_d(p) = \begin{cases} 1, if\ p \in I \\ 0, otherwise \end{cases}$, $0 \leq t < 1$ for all $d \in D$.

Proof: Let H is a prime tideal of F. Let $(g,F_2,\Gamma),(h,F_3,\Gamma)\, and\, (j,F_4,\Gamma)$ be FS tideals over F where $(g,F_2,\Gamma)o(h,F_3,\Gamma)o(j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$ ------ (1)

Suppose $(g,F_2,\Gamma) \not\subseteq (f,F_1,\Gamma)$, $(h,F_3,\Gamma) \not\subseteq (f,F_1,\Gamma)$, $(j,F_4,\Gamma) \not\subseteq (f,F_1,\Gamma)$ Then

$$g_y(p) > f_x(p), h_z(q) > f_x(q),\; j_w(r) > f_x(r)\;\; for\ some\ p,q,r \in F, \alpha,\beta \in \Gamma.$$

Obviously $f_x(p) \neq 1, f_x(q) \neq 1\, and\, f_x(r) \neq 1$. Hence $f_x(p) = t, f_x(q) = t\, and\, f_x(r) = t$

$$\Rightarrow p,q,r \notin H \Rightarrow [p\alpha q\beta r] \notin H,\; \alpha,\beta \in \Gamma --(2)$$

$$\begin{aligned} f_x(v) = f_x([p\alpha q\beta r]) &\geq \bigvee_{v=[p\alpha q\beta r]} g_y(p) \wedge h_z(q) \wedge j_w(r) \\ &\geq g_y(p) \wedge h_z(q) \wedge j_w(r) \\ &\succ t \end{aligned}$$

$\Rightarrow f_x([p\alpha q\beta r]) = 1 \Rightarrow [p\alpha q\beta r] \in H$ Which is a contradiction to (2) and so $(g,F_2,\Gamma) \subseteq (f,F_1,\Gamma) or (h,F_3,\Gamma) \subseteq (f,F_1,\Gamma) or (j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$. Therefore (f,F_1,Γ) is a FSP tideal over F.

Now, suppose (f,F_1,Γ) be a FSP tideal over F. Let P, Q, R be the tideals of F such that $[P\Gamma Q\Gamma R] \subseteq H$. Let $(g,F_2,\Gamma),(h,F_3,\Gamma) and (j,F_4,\Gamma)$ be FS sets over F defined by $g_b(p) = \begin{cases} 1, if\ p \in P \\ 0,\ otherwise \end{cases}$ $h_c(q) = \begin{cases} 1, if\ q \in Q \\ 0,\ otherwise \end{cases}$ $j_d(r) = \begin{cases} 1, if\ r \in R \\ 0,\ otherwise \end{cases}$ for every b ∈ F_2, c ∈ F_3 and d ∈ F_4. Then $(g,F_2,\Gamma),(h,F_3,\Gamma) and (j,F_4,\Gamma)$ are FS tideals over F and it can be established that $(g,F_2,\Gamma) o (h,F_3,\Gamma) o (j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$ --(3)

$\Rightarrow (g,F_2,\Gamma) \subseteq (f,F_1,\Gamma) or\ (h,F_3,\Gamma) \subseteq (f,F_1,\Gamma) or\ (j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$ ---(4)

Suppose $P \not\subseteq H, Q \not\subseteq H, R \not\subseteq H$, then ∃ p ∈ P, q ∈Q, r ∈ R such that $p \notin H, q \notin H, r \notin H$ $\Rightarrow g_b(p) = 1 = h_c(q) = j_d(r) \Rightarrow g_b(p) = 1 \succ t = f_a(p) \Rightarrow (g,F_2,\Gamma) \not\subseteq (f,F_1,\Gamma)$. Similarly, $(h,F_3,\Gamma) \not\subseteq (f,F_1,\Gamma) (j,F_4,\Gamma) \not\subseteq (f,F_1,\Gamma)$ contradicting (4). Hence H is a prime tideal of F.

The particular case (ii) is obtained from (i) by putting H = I and t = 0.

Th 3.16: A FS set (f,F_1,Γ) over F is a FSP tideal over F iff for every f_x, where x ∈ F_1, $(f_x)_t, t \in \operatorname{Im} f_x$ is a prime tideal of F.

Proof: Let (f,F_1,Γ) be a FSP tideal over F. Let P, Q, R be tideals of F such that $[P\Gamma Q\Gamma R] \subseteq (f_x)_t$. Let $(g,F_2,\Gamma),(h,F_3,\Gamma) and (j,F_4,\Gamma)$ be FS sets over F and $g_y(p) = \begin{cases} t, if\ p \in P \\ 0,\ otherwise \end{cases}$, $h_z(q) = \begin{cases} t, if\ q \in Q \\ 0,\ otherwise \end{cases}$ $j_w(r) = \begin{cases} t, if\ r \in R \\ 0,\ otherwise \end{cases}$

Where $t \in \operatorname{Im} f_x$, $F_2, F_3, F_4 \subseteq F_1 \subseteq E$ and y ∈ F_2, z ∈ F_3, w ∈ F_4. Then (g,F_2,Γ), $(h,F_3,\Gamma),(j,F_4,\Gamma)$ are FS tideals over F and also $(g,F_2,\Gamma) o (h,F_3,\Gamma) o (j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$. since (f,F_1,Γ) is a FSP tideal over F.⇒ $(g,F_2,\Gamma) \subseteq (f,F_1,\Gamma)$ or $(h,F_3,\Gamma) \subseteq (f,F_1,\Gamma)$ or $(j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$ ----- (1). Suppose $P \not\subseteq (f_x)_t$. Then ∃ p ∈ P but $p \notin (f_x)_t \Rightarrow f_x(p) < t$. Now $g_y(p) = t$ (∵ p ∈ P) > $f_x(p)$. ⇒ $(g,F_2,\Gamma) \not\subseteq (f,F_1,\Gamma)$. Similarly $(h,F_3,\Gamma) \not\subseteq (f,F_1,\Gamma)$, $(j,F_4,\Gamma) \not\subseteq (f,F_1,\Gamma)$ contradicting to (1). Hence $(f_x)_t$ is a prime tideal of F.

Now, let $(f_x)_t$ is a prime tideal of F. Let F_2, F_3, F_4 ⊆ F_1 ⊆ E such that (g,F_2,Γ), $(h,F_3,\Gamma) and (j,F_4,\Gamma)$ are FS tideals and $(g,F_2,\Gamma) o (h,F_3,\Gamma) o (j,F_4,\Gamma) \subseteq (f,F_1,\Gamma)$.

Suppose $(g,F_2,\Gamma) \not\subseteq (f,F_1,\Gamma)$, $(h,F_3,\Gamma) \not\subseteq (f,F_1,\Gamma)$, $(j,F_4,\Gamma) \not\subseteq (f,F_1,\Gamma)$----(2)

Then $g_y(p) > f_x(p), h_z(q) > f_x(q), j_w(r) > f_x(r)\ for\ p,q,r \in F, \alpha, \beta \in \Gamma$.

Obviously $f_x(p) \neq 1, f_x(q) \neq 1, f_x(r) \neq 1$. Let $f_x(p) = s = f_x(q) = f_x(r)$.

$\Rightarrow g_y(p) > s, h_z(q) s, j_w(r) > s.$

$$But\ f_x(v) \geq \bigvee_{v=[p\alpha q\beta r]} g_y(p) \wedge h_z(q) \wedge j_w(r)$$
$$\geq g_y(p) \wedge h_z(q) \wedge j_w(r)$$
$$> t$$

$\Rightarrow [p\alpha q\beta r] \in (f_x)_s \Rightarrow \left[(g_y)_s \Gamma (h_z)_s \Gamma (j_w)_s\right] \subseteq (f_x)_s$. Which is a prime tideal.

$\therefore (g_y)_s \subseteq (f_x)_s\ or (h_z)_s \subseteq (f_x)_s\ or (j_w)_s \subseteq (f_x)_s \Rightarrow p \in (g_y)_s \Rightarrow p \in (f_x)_s$

$\Rightarrow g_y(p) \geq s \Rightarrow f_x(p) \geq s.$ Which contradicts (2). Hence $(g, F_2, \Gamma) \subseteq (f, F_1, \Gamma)$ or $(h, F_3, \Gamma) \subseteq (f, F_1, \Gamma)$ or $(j, F_4, \Gamma) \subseteq (f, F_1, \Gamma)$. Hence (f, F_1, Γ) is a FSP tideal over F.

Remark 3.17: If an tideal H of a CP tideal of a FSTΓ-SR F then H be a prime tideal of F.

Th 3.18: Let (f, F_1, Γ) be a FSS over F, then for $x \in F_1, N = \{p \in F / f_x(p) = f_x(0)\}$ is a CP tideal of F if (f, F_1, Γ) is a FSCP tideal over F.

Proof: Let (f, F_1, Γ) be a FSCP tideal over F. Then N is a tideal over F.

Suppose $[p\alpha q\beta r] \in N\ for\ \alpha, \beta \in \Gamma$, then $f_x([p\alpha q\beta r]) = f_x(0)$. But $f_x([p\alpha q\beta r])$ $\leq \max\{f_x(p), f_x(q), f_x(r)\} \Rightarrow f_x(0) \leq \max\{f_x(p), f_x(q), f_x(r)\} \Rightarrow f_x(0) \leq f_x(p)$ $or\ f_x(0) \leq f_x(q) or\ f_x(0) \leq f_x(r)$. But $f_x(0) \geq f_x(p)\ for\ p \in F$. Hence $f_x(p) = f_x(0)$ $or\ f_x(q) = f_x(0) or\ f_x(q) = f_x(0). \Rightarrow p \in N\ or\ q \in N\ or\ r \in N$. So N is a prime tideal

Note 3.19: The converse of the Th 3.18 holds if $|\mathrm{Im}\ f_x| = 2, x \in F_1$.

Th 3.20: Let (f, F_1, Γ) is a FS set over F. Let $|\mathrm{Im}\ f_x| = 2\ and\ f_x(0) = 1$ and if $N = \{p \in F / f_x(p) = 1\}$ is a CP tideal then (f, F_1, Γ) be a FSCP tideal over F.

Proof: Proof is similar to Th 3.14.

Th 3.21: Let (f, F_1, Γ) is a FS set over F, then (f, F_1, Γ) be a FSCP tideal over F iff for $x \in F_1, |\mathrm{Im}\ f_x| = 2$, $f_x(0) = 1$ and $N = \{p \in F / f_x(p) = 1\}$ is a CP tideal of F.

Lemma 3.22: A non empty subset H of F is an tideal of F iff (f, F_1, Γ) is a FS tideal over F, where $f : F_1 \to [0,1]$ is defined by $f_x(p) = \begin{cases} s, & if\ p \in H \\ t, if & p \in F - H \end{cases}$ $Where\ r > t\ for\ all\ x \in F_1$.

Th 3.23: A non empty subset H of F is a CP tideal of F. If (f, F_1, Γ) be a FSCP tideal over F. where $f : F_1 \to [0,1]$ is defined by $f_x(p) = \begin{cases} s, if & p \in H \\ t, if & p \notin H \end{cases}$ $Where\ s > t\ and\ t \in [0,1]\ for\ all\ x \in F_1$,

Proof: Let (f, F_1, Γ) be a FSCP tideal over F, then by lemma 3.22, H is a tideal of F. Let $\alpha, \beta \in \Gamma$ and $[p\alpha q\beta r] \in H, then\ f_x([p\alpha q\beta r]) = s \Rightarrow \max\{f_x(p), f_x(q), f_x(r)\} \geq s$ $\Rightarrow either\ f_x(p) \geq s\ or\ f_x(q) \geq s\ or\ f_x(r) \geq s. \Rightarrow p \in H\ or\ q \in H\ or\ r \in H$. So H is a CP tideal of F.

Now, Let H is a CP tideal. By Lemma 3.22, (f, F_1, Γ) is a FS tideal over F. Suppose $f_x([p\alpha q\beta r]) \succ \max\{f_x(p), f_x(q), f_x(r)\}$, then by definition of (f, F_1, Γ), $f_x([p\alpha q\beta r]) = s$ and $f_x(p) = t = f_x(q) = f_x(r)$. *Thus* $[p\alpha q\beta r] \in H$ *but* $p \notin H, q \notin H$, $r \notin H$. which is a contradiction to H is a CP tideal of F. Hence (f, F_1, Γ) is a FSCP tideal over F.

Corollary 3.24: A subset $H(\neq \phi)$ of F is a CP tideal iff the characteristic function (Ψ_H, D) is a FSCP tideal over F, where $(\Psi_H)_d : D \to H^F$ is defined by $\Psi_H(p) = \begin{cases} 1, if\ p \in H \\ 0, if\ p \notin H \end{cases}$ *for all* $d \in D$.

Proof: In the Th 3.23 take s =1, t = 0 then the proof follows by Th 3.23.

Th 3.25: Let $F_1 \subseteq E(Parameter\ set)$ of an FSTΓ-SR F. Then a FS set (f, F_1, Γ) over F is a FSCP tideal $\Rightarrow$ *for each* f_x, *where* $x \in F_1, (f_x)_t, t \in \operatorname{Im} f_x$ is a CP tideal of F.

Th 3.26: Let H be a CP tideal of F. For t ∈ (0, 1) ∃ a FSCP tideal (f, F_1, Γ) over F such that $(f_x)_t = H$ for each $x \in F_1$.

Proof: Let H be a CP tideal of F. Let t ∈ (0, 1). Define (f, F_1, Γ) over F by $f_x(p) = \begin{cases} t, if\ p \in H \\ 0, if\ p \notin H \end{cases}$ Then by Th 3.11 (f, F_1, Γ) is a FS tideal over F. If (f, F_1, Γ) is not FSCP tideal then $\exists\ p, q, r \in F$, $\alpha, \beta \in \Gamma \ni f_x([p\alpha q\beta r]) > \max\{f_x(p), f_x(q), f_x(r)\}$. Now, we have $f_x(p) = 0 = f_x(q) = f_x(r)$ *and* $f_a([p\alpha q\beta r]) = t$. Thus, $[p\alpha q\beta r] \in H$ *but* $p \notin H, q \notin H, r \notin H$. which contradicts that H is a CP tideal of F. Hence (f, F_1, Γ) is a FSCP tideal over F. obviously, $(f_x)_t = H$ *for each* $x \in F_1$.

Th 3.27: Let (f, F_1, Γ) is a FSCP tideal over F with $|\operatorname{Im} f_x| = 2, f_x(0) = 1$ for all a ∈ F_1, then (f, F_1, Γ) is a FSP tideal over F.

Proof: Let (f, F_1, Γ) be a FSCP tideal over F such that $|\operatorname{Im} f_x| = 2, f_x(0) = 1$, hence by Remark 3.17, $N = \{p \in F / f_x(p) = 1\}$ is a CP tideal. By Th 3.20, T is a prime tideal of F. Hence by Th 3.21, (f, F_1, Γ) is a FSP tideal over F

4. CONCLUSION

In this paper we introduced the notations of FSP tideals and FSCP tideals in Γ-Semirings and characterized them.

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