Fuzzy Soft Ternary Γ-Semirings-III

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ABSTRACT

The notations of fuzzy soft ternary Γ-semiring (FSTΓ-SR), Fuzzy soft TΓ-ideal (FSTΓI) and fuzzy soft interior TΓ-ideal (FSITΓI) over a ternary Γ-semiring (TΓ-SR), and study their algebraic properties and relations between them are introduced.

Keywords: TΓ-Semiring, Fuzzy soft TΓ-Semiring, Fuzzy soft TΓ-ideal, fuzzy soft interior TΓ-ideal.

1. INTRODUCTION

The notations of TΓ-Semiring introduced by Madhusudhana Rao and SajaniLavanya [3] in the year 2015. Earlier in 1932 Lehmer [2] was introduced the concept of ternary algebraic system. In 1971 the algebraic structures fuzzification was introduced by A. Rosenfeld [4].

2. PRELIMINARIES

Def 2.1: Let N, Γ be two positive commutative semigroups, N is said to be a TΓ-SR if there exit $NX\Gamma XNX\Gamma XN \to N$ as $(n_1,\alpha,n_2,\beta,n_3) \to [n_1\alpha n_2\beta n_3]$ satisfying the conditions.

$i)\left[[n_1\alpha n_2\beta n_3]\gamma n_4\delta n_5\right] = \left[n_1\alpha[n_2\beta n_3\gamma n_4]\delta n_5\right] = \left[n_1\alpha n_2\beta[n_3\gamma n_4\delta n_5]\right]$

$ii)\left[(n_1+n_2)\alpha n_3\beta n_4\right] = [n_1\alpha n_3\beta n_4]+[n_2\alpha n_3\beta n_4], iii)\left[n_1\alpha(n_2+n_3)\beta n_4\right] = [n_1\alpha n_2\beta n_4]+[n_1\alpha n_3\beta n_4]$

$iv)\left[n_1\alpha n_2\beta(n_3+n_4)\right] = [n_1\alpha n_2\beta n_3]+[n_1\alpha n_2\beta n_4],\ for\ all\ n_1,n_2,n_3,n_4 \in N, \alpha,\beta,\gamma,\delta \in \Gamma$

Def 2.2: Let N be a TΓ-SR. E be a parameter set and $U \subseteq E$. Let f be a mapping as $f : U \to [0,1]^N$ where $[0,1]^N$ is the collection of all fuzzy subsets(FSSs) of N, then (f,U,Γ) is called a FSTΓ-SR over N iff for each $p \in U, f(p) = f_p$ is the FTΓ-SSR (fuzzy ternary Γ-Subsemiring) of V. i.e., $i) f_p(j+k) \geq \min\{f_p(j), f_p(k)\}$

$ii) f_p(j\gamma k\delta l) \geq \min\{f_p(j), f_p(k), f_p(l)\}$ $for\ all\ j,k,l \in N\ and\ \gamma,\delta \in \Gamma$

Def 2.3: Let N be a TΓ-SR. E be a parameter set and $U \subseteq E$. If f be a mapping as $f : U \to [0,1]^N$ where $[0,1]^N$ is the collection of all FSSs of N, then (f,U,Γ) is known as a FSLTΓI (FSMTΓI, FSRTΓI) (fuzzy soft t-left(t-lateral, t-right) **Γ**-ideal) over N iff for each $p \in U$, the corresponding FSS $f_p : N \to [0,1]$ is a FLTI (FMTI, FRTI) of N .i.e.,

$i) f_p(j+k) \geq \min\{f_p(j), f_p(k)\}$ $ii) f_p(j\gamma k\delta l) \geq f_p(l)(f_p(k), f_p(j))$

$for\ all\ j,k,l \in N\ and\ \gamma,\delta \in \Gamma.$

Def 2.4: Let N be a TΓ-SR. E be a parameter set and $U \subseteq E$. Let f be a mapping as $f : U \to [0,1]^N$ where $[0,1]^N$ is the collection of all FSSs of N, then (f,U,Γ) is called a *FSTΓI* over N iff for each $p \in U$, the corresponding FSS $f_p : N \to [0,1]$ is a fuzzy ideal of N.

i.e., i) $i) f_p(j+k) \geq \min\{f_p(j), f_p(k)\}$

ii) $ii) f_p(j\gamma k\delta l) \geq \max\{f_p(j), f_p(k), f_p(l)\}$ $for\ all\ j,k,l \in N\ and\ \gamma,\delta \in \Gamma.$

3. MAIN RESULTS

Def 3.1: Let N be a TΓ-SR. E be a parameter set and $U \subseteq E$.Let f be a mapping as $f : U \to [0,1]^N$ where $[0,1]^N$ is the collection of all FSSs of N, then (f,U,Γ) is called a *FSITΓI* over N iff for each $p \in U$, the corresponding FSS $f_p : N \to [0,1]$ is a FITΓI of N.

i.e., $i) f_p(j+k) \geq \min\{f_p(j), f_p(k)\}$ $ii) f_p(j\gamma k\delta l) \geq \min\{f_p(j), f_p(k), f_p(l)\}$

$iii) f_p(j\alpha k\beta l\gamma s\delta t) \geq \min\{f_p(k), f_p(s)\}$ for all $j,k,l,s,t \in N\ and\ \alpha,\beta,\gamma,\delta \in \Gamma$

Def 3.2: Let $(f,U,\Gamma)\ and\ (g,V,\Gamma)$ be Fuzzy Soft (FS) Sets over a TΓ-SR N. The intersection of FS Sets is denoted by $(f,U,\Gamma) \cap (g,V,\Gamma) = (h,W,\Gamma)$ where $W = U \cup V$ is defined as

$$j_p = \begin{cases} f_p & ,if\ p \in U - V \\ g_p & ,if\ p \in V - U \\ f_p \cap g_p & ,if\ p \in U \cap V \end{cases} \quad for\ all\ p \in U \cup V,\ x \in N$$

Def 3.3: Let $(f,U,\Gamma)\ and\ (g,V,\Gamma)$ be FS Sets over a TΓ-SR N. The union of FS Sets is denoted by $(f,U,\Gamma) \cup (g,V,\Gamma) = (\mathrm{j},W,\Gamma)$ where $W = U \cup V$ is defined as

$$j_P = \begin{cases} f_p & ,if\ p \in U - V \\ g_p & ,if\ p \in V - U \\ f_p \cup g_p & ,if\ p \in U \cap V \end{cases} \quad for\ all\ p \in U \cup V,\ x \in N.$$

Def 3.4: Let $(f,U,\Gamma)\, and\, (g,V,\Gamma)$ be FS Sets over a TΓ-SR N, then $(f,U,\Gamma)\wedge(g,V,\Gamma)=(h,W,\Gamma)$ where $E=UXV, h_p(x)=\min\{f_u(x),g_v(x)\}$ for all $p=(u,v)\in UXV\, and\, x\in N.$

Def 3.5: Let $(f,U,\Gamma)\, and\, (g,V,\Gamma)$ be FS Sets over a TΓ-SR N, then $(f,U,\Gamma)\vee(g,V,\Gamma)=(h,W,\Gamma)$ where $W=UXV, W_P(x)=\max\{f_u(x),g_v(x)\}$ for all $p=(u,v)\in UXV\, and\, x\in N.$

Def 3.6: Let $(f,\mathrm{U},\Gamma),(g,\mathrm{V},\Gamma)\, and\, (h,\mathrm{W},\Gamma)$ are FSTΓIs of a TΓ-SR N, then the composition is defined as $((fogoh),X,\Gamma)\, where\, P=U\cup V\cup W$ and

$$(fogoh)_p(x)=\begin{cases} f_p(x) & ,if\ p\in U-(V\cup W)\\ g_p(x) & ,if\ p\in V-(U\cup W)\\ h_p(x) & ,if\ p\in W-(U\cup V)\\ Sup\{\min\{f_p(u),f_p(v),f_p(w)\}\}, & if\ p\in U\cap V\cap W, p=a\alpha b\beta c\end{cases}$$

for all $p\in U\cup V\cup W, x\in N\, and\, \alpha,\beta\in\Gamma$.

Th 3.7: Let $(f,U,\Gamma)\, and\, (g,V,\Gamma)$ be FSTΓ-SRs over a TΓ-SR N, then $(f,U,\Gamma)\cup(g,V,\Gamma)$ is a FSTΓ-SR over N.

Proof: Let $(f,U,\Gamma)\cup(g,V,\Gamma)=(h,W,\Gamma)$ where $W=U\cup V$

Case (i) If $U\cap V=\phi$. Let $p\in P=U\cup V\, then\, p\in U\, or\, p\in V$

$If\ p\in U\, then\, h_p=f_p, if\ p\in V\, then\, h_p=g_p$

Here $(f,U,\Gamma)\, and\, (g,V,\Gamma)$ are FSTΓ-SRs over N Implies h_p is a FSTΓ-SSR.

Case (ii) If $U\cap V\neq\phi$

$If\ p\in U-V\, then\, h_p=f_p$ Implies h_p is a FTΓ-SSR.

$If\ p\in V-U\, then\, h_p=g_p$ Implies h_p is a FTΓ-SSR.

$If\ p\in U\cap V\, then\, h_p=f_p\cup g_p. Let\, n_1,n_2,n_3\in N,\alpha,\beta\in\Gamma$

$$\begin{aligned} then\ h_p(n_1+n_2) &= (f_p\cup g_p)(n_1+n_2)\\ &=\max\{f_p(n_1+n_2),g_p(n_1+n_2)\}\\ &\geq\max\{\min\{f_p(n_1),f_p(n_2)\},\min\{g_p(n_1),g_p(n_2)\}\}\\ &=\min\{\max\{f_p(n_1),f_p(n_2)\},\max\{g_p(n_1),g_p(n_2)\}\}\\ &=\min\{\max\{f_p(n_1),g_p(n_1)\},\max\{f_p(n_2),g_p(n_2)\}\}\end{aligned}$$

$$= \min\left\{\left(f_p \cup g_p\right)(n_1), \left(f_p \cup g_p\right)(n_2)\right\}$$
$$= \min\left\{h_p(n_1), h_p(n_2)\right\}$$

$Now\ h_p(n_1 \alpha n_2 \beta n_3) = \left(f_p \cup g_p\right)(n_1 \alpha n_2 \beta n_3)$

$$= \max\left\{f_p(n_1 \alpha n_2 \beta n_3), g_p(n_1 \alpha n_2 \beta n_3)\right\}$$
$$\geq \max\left\{\min\left\{f_p(n_1), f_p(n_2), f_p(n_3)\right\}, \min\left\{g_p(n_1), g_p(n_2), g_p(n_3)\right\}\right\}$$
$$= \min\left\{\max\left\{f_p(n_1), f_p(n_2), f_p(n_3)\right\}, \min\left\{g_p(n_1), g_p(n_2), g_p(n_3)\right\}\right\}$$
$$= \min\left\{\max\left\{f_p(n_1), g_p(n_1)\right\}, \max\left\{f_p(n_2), g_p(n_2)\right\}, \max\left\{f_p(n_3), g_p(n_3)\right\}\right\}$$
$$= \min\left\{\left(f_p \cup g_p\right)(n_1), \left(f_p \cup g_p\right)(n_2), \left(f_p \cup g_p\right)(n_3)\right\}$$
$$= \min\left\{h_d(n_1), h_d(n_2), h_d(n_3)\right\}$$

Therefore h_p is a FTΓ-SSR of N Implies $(f, U, \Gamma) \cup (g, V, \Gamma)$ is a FSTΓ-SR over N.

Th 3.8: Let $(f, \mathrm{U}, \Gamma), (g, \mathrm{V}, \Gamma)\ and\ (h, \mathrm{W}, \Gamma)$ are FSTΓ-SRs over a TΓ-SR N, then $(f, U, \Gamma) \cap (g, V, \Gamma) \cap (h, W, \Gamma)$ is a FSTΓ-SR over N.

Th 3.9: Let $(f, \mathrm{U}, \Gamma), (g, \mathrm{V}, \Gamma)\ and\ (h, \mathrm{W}, \Gamma)$ are FSTΓ-SRs over a TΓ-SR N, then $(f, U, \Gamma) \wedge (g, V, \Gamma) \wedge (h, W, \Gamma)$ is a FSTΓ-SR over N.

Th 3.10: Let $(f, \mathrm{U}, \Gamma)\ and\ (g, \mathrm{V}, \Gamma)$ be FSTΓ-SRs over a TΓ-SR N, then $(f, \mathrm{U}, \Gamma) \vee (g, \mathrm{V}, \Gamma)$ is a FSTΓ-SR over N.

Def 3.11: Let $(f, \mathrm{U}, \Gamma)\ and\ (g, \mathrm{V}, \Gamma)$ be FSTΓ-SRs over a TΓ-SR N, then (g, V, Γ) is a FSTΓ-SSR of (f, U, Γ) if $i) V \subseteq U \quad ii) g_v(x) \leq f_u(x), \forall x \in N.$

Th 3.12: Let N be a TΓ-SR and E be a parameter set and $U \subseteq E.$ If (f, U, Γ) is a FSLTΓI (FSMTΓI , FSRTΓI) over N, then if for each $\left(\chi_n o f_u o \chi_n \subseteq f_u, f_u o \chi_n o \chi_n \subseteq f_u\right)$ where χ_n stands for characteristic function of N.

Proof: Suppose (f, U, Γ) is a FSLTΓI over N, then for each $u \in U, f_u$ is a FLTΓI of N.

$Let\ n \in N, then \left(\chi_n o \chi_n o f_u\right)(u) = \underset{n \leq n_1 \alpha n_2 \beta n_3}{Sup} \left\{\min\left\{\chi_n(n_1), \chi_n(n_2). \chi_n(n_3)\right\}\right\}$

$$= \underset{n \leq n_1 \alpha n_2 \beta n_3}{Sup} \left\{f_u(n)\right\}$$
$$\leq f_u(n_1 \alpha n_2 \beta n_3)$$
$$= f_u(n)$$

If N cannot be expressed as $n \leq n_1 \alpha n_2 \beta n_3, where\ n_1, n_2, n_3 \in N\ and\ \alpha, \beta \in \Gamma\ then$ $\left(\chi_n o \chi_n o f_u\right)(n) = 0 \leq f_u(n).$ lly we can prove the FSMTΓI and FSRTΓI over N.

Th 3.13: Let $(f, \mathrm{U}, \Gamma), (g, \mathrm{V}, \Gamma)\ and\ (h, \mathrm{W}, \Gamma)$ be FSITΓIs over a TΓ-SR N, then $(f, \mathrm{U}, \Gamma) \cap (g, \mathrm{V}, \Gamma) \cap (h, \mathrm{W}, \Gamma)$ is a FSITΓI over N.

Proof: By definition 3. 2, $(f, \mathrm{U}, \Gamma) \cap (g, \mathrm{V}, \Gamma) \cap (h, \mathrm{W}, \Gamma) = (i, P, \Gamma)$

$where\ P = U \cup V \cup W$

Case (i): $If\ p \in U-(V \cup W) then\, i_p = f_p$ implies i_p is a FSITΓI over N, because (f, U, Γ) is a FSITΓI over N.

Case (ii): $If\ p \in V-(U \cup W) then\, i_p = g_p$ implies i_p is a FSITΓI over N, because (g, V, Γ) is a FSITΓI over N.

Case (iii): $If\ p \in W-(U \cup V) then\, i_p = h_p$ implies i_p is a FSITΓI over N, because (h, W, Γ) is a FSITΓI over N.

Case (iv): $If\ p \in U \cap V \cap W\ and\ x, y, z, s, t \in N,\ \alpha, \beta, \gamma \in \Gamma\, then\, i_p = f_p \cap g_p \cap h_p$ and

$$
\begin{aligned}
i_p(x+y) &= \min\{f_p(x+y), g_p(x+y), h_p(x+y)\} \\
&\geq \min\{\min\{f_p(x), f_p(y)\}, \min\{g_p(x), g_p(y), \min\{h_p(x), h_p(y)\}\}\} \\
&= \min\{\min\{f_p(x), g_p(x), h_p(x)\}, \min\{f_p(y), g_p(y), h_p(y)\}\} \\
&= \min\{(f_p \cap g_p \cap h_p)(x), (f_p \cap g_p \cap h_p)(y)\} \\
&= \min\{i_p(x), i_p(y)\}
\end{aligned}
$$

$$
\begin{aligned}
i_p(x\alpha s\beta y\gamma t\delta z) &= \mathrm{mi}\,v\{f_p(x\alpha s\beta y\gamma t\delta z), g_p(x\alpha s\beta y\gamma t\delta z), h_p(x\alpha s\beta y\gamma t\delta z)\} \\
&\geq \min\{\min\{\{f_p(s), f_p(t)\}, \{g_p(s), g_p(t)\}, \{h_p(s), h_p(t)\}\}\} \\
&= \min\{\min\{\{f_p(s), g_p(s), h_p(s)\}, \{f_p(t), g_p(t), h_p(t)\}\}\} \\
&= \min\{(f_p \cap g_p \cap h_p)(s), (f_p \cap g_p \cap h_p)(t)\} \\
&= \min\{i_p(s), i_p(t)\}
\end{aligned}
$$

Hence i_p is a FITΓI of N. Thus $(f, \mathrm{U}, \Gamma) \cap (g, \mathrm{V}, \Gamma) \cap (h, \mathrm{W}, \Gamma)$ is a FSITΓI over N.

Th 3.14: Let $(f, \mathrm{U}, \Gamma)\ and\ (g, \mathrm{V}, \Gamma)$ be FSTΓIs over a TΓ-SR N, then $(f, \mathrm{U}, \Gamma) \cup (g, \mathrm{V}, \Gamma)$ is a FSTΓI over N.

Proof: Suppose $(f, \mathrm{U}, \Gamma) \cup (g, \mathrm{V}, \Gamma) = (h, W, \Gamma)\, where\, W = U \cup V\ and$

$$
h_p = \begin{cases} f_p & , if\ p \in U-V \\ g_p & , if\ p \in V-U \\ f_p \cup g_p & , if\ p \in U \cap V \end{cases}
$$

Case (i): $If\ p \in U-V\, then\, h_p = f_p$ implies h_p is a FTΓI of N. Since (f, U, Γ) is a FSTΓI over N.

Case (ii): $If\ p \in V-U\, then\, h_p = g_p$ implies h_p is a FTΓI of N. Since (g, V, Γ) is a FSTΓI over N.

Case (iii): $If\ p \in U \cap V\ then\ for\ all\ x, y, z \in N, \alpha, \beta \in \Gamma.$

$$
h_p(x) = (f_p \cup g_p)(x) = \max\{f_p(x), g_p(x)\}
$$

$$
\begin{aligned}
h_p(x+y) &= \max\{f_p(x+y), g_p(x+y)\} \\
&\geq \max\{\min\{f_p(x), f_p(y)\}, \min\{g_p(x), g_p(y)\}\} \\
&= \min\{\max\{f_p(x), g_p(x)\}, \max\{f_p(y), g_p(y)\}\} \\
&= \min\{(f_p \cup g_p)(x), (f_p \cup g_p)(y)\} \\
&= \min\{h_c(x), h_c(y)\}
\end{aligned}
$$

$$Now\, h_p(x\alpha y\beta z) = (f_p \cup g_p)(x\alpha y\beta z)$$
$$= \max\{f_p(x\alpha y\beta z), g_p(x\alpha y\beta z)\}$$
$$\geq \max\{\max\{f_p(x), f_p(y), f_p(z)\}, \max\{g_p(x), g_p(y), g_p(z)\}\}$$
$$= \max\{\max\{f_p(x), g_p(x)\}, \max\{f_p(y), g_p(y)\}, \max\{f_p(z), g_p(z)\}\}$$

Hence h_p is a FTΓI of N. Therefore (h, W, Γ) is a FSTΓI over N.

Th 3.15: Let (f, U, Γ) *and* (g, V, Γ) be FSITΓIs over a TΓ-SR N, then $(f, \text{U}, \Gamma) \cup (g, \text{V}, \Gamma)$ is a FSITΓI over N.

Proof: Suppose $(f, U, \Gamma) \cup (g, V, \Gamma) = (h, W, \Gamma)\, where\, W = UXV\ and$

$$h_p = \begin{cases} f_p & , if\ p \in U - V \\ g_p & , if\ p \in V - U \\ f_p \cup g_p & , if\ p \in U \cap V \end{cases}$$

Case (i): $If\ p \in U - V\ then\, h_p = f_p$ implies h_p is a FITΓI of N. Since (f, U, Γ) is a FSITΓI over N.

Case (ii): $If\ p \in V - U\ then\, h_p = g_p$ implies h_p is a FITΓI of N. Since (g, V, Γ) is a FSITΓI over N.

Case (iii): $If\ p \in U \cap V\ then\ for\ all\ x, y, z, s, t \in N, \alpha, \beta, \gamma, \delta \in \Gamma.$

$$h_p(x) = (f_p \cup g_p)(x) = \max\{f_p(x), g_p(x)\}$$
$$h_p(x+y) = \max\{f_p(x+y), g_p(x+y)\}$$
$$\geq \max\{\min\{f_p(x), f_p(y)\}, \min\{g_p(x), g_p(y)\}\}$$
$$= \min\{\max\{f_p(x), g_p(x)\}, \max\{f_p(y), g_p(y)\}\}$$
$$= \min\{(f_p \cup g_p)(x), (f_p \cup g_p)(y)\}$$
$$= \min\{h_p(x), h_p(y)\}$$
$$h_p(x\alpha s\beta y\gamma t\delta z) = \min\{f_p(x\alpha s\beta y\gamma t\delta z), g_p(x\alpha s\beta y\gamma t\delta z), h_p(x\alpha s\beta y\gamma t\delta z)\}$$
$$\geq \min\{\min\{f_p(s), f_p(t)\}, \min\{g_p(s), g_p(t)\}\}$$
$$= \min\{\min\{f_p(s), g_p(s)\}, \min\{f_p(t), g_p(t)\}\}$$
$$= \min\{(f_p \cap g_p)(s), (f_p \cap g_p)(t)\}$$
$$= \min\{h_p(s), h_p(t)\}$$

Hence h_p is a FITΓI of N. Therefore (h, W, Γ) is a FSITΓI over N.

Th 3.16: Let (f, U, Γ) *and* (g, V, Γ) be FSTΓIs over a TΓ-SR N, then $(f, \text{U}, \Gamma) \wedge (g, \text{V}, \Gamma)$ is a FSTΓI over N.

Proof: Suppose $(f, U, \Gamma) \cup (g, V, \Gamma) = (h, W, \Gamma)\, where\ W = UXV$

$$Let\ p=(u,v)\in W=UXV\ and\ x,y,z\in N,\alpha,\beta\in\Gamma$$
$$then\ h_p(x+y)=f_u(x+y)\wedge g_v(x+y)$$
$$=\min\{f_u(x+y),g_v(x+y)\}$$
$$\geq\min\{\min\{f_u(x),f_u(y)\},\min\{g_v(x)\},g_v(y)\}$$
$$=\min\{\min\{f_u(x),g_v(x)\},\min\{f_u(y),g_v(y)\}\}$$
$$=\min\{(f_u\wedge g_v)(x),(f_u\wedge g_v)(y)\}$$
$$\min\{h_p(x),h_p(y)\}$$
$$Now\ h_p(x\alpha y\beta z)=f_u(x\alpha y\beta z)\wedge g_v(x\alpha y\beta z)$$
$$=\min\{f_u(x\alpha y\beta z),g_v(x\alpha y\beta z)\}$$
$$\geq\min\{\max\{f_u(x),f_u(y),f_u(z)\},\max\{g_v(x),g_v(y),g_v(z)\}\}$$
$$=\max\{\min\{f_u(x),f_u(y),f_u(z)\},\min\{g_v(x),g_v(y),g_v(z)\}\}$$

Hence h_p is a FITΓI of N. Therefore (h,W,Γ) is a FSITΓI over N.

Th 3.17: Let (f,U,Γ) *and* (g,V,Γ) be FSITΓIs over a TΓ-SR N, then $(f,U,\Gamma)\wedge(g,V,\Gamma)$ is a FSITΓI over a TΓ-SR N.

Proof: Suppose $(f,U,\Gamma)\wedge(g,V,\Gamma)=(h,W,\Gamma)\ where\ W=UXV$

$$Let\ P=(u,v)\in W=UXV\ and\ x,y,z,s,t\in V,\alpha,\beta,\gamma,\delta\in\Gamma.$$
$$then\ h_p(x+y)=f_u(x+y)\wedge g_v(x+y)$$
$$=\min\{f_u(x+y),g_v(x+y)\}$$
$$\geq\min\{\min\{f_u(x),f_u(y)\},\min\{g_v(x),g_v(y)\}\}$$
$$=\min\{\min\{f_u(x),g_v(x)\},\min\{f_u(y),g_v(y)\}\}$$
$$=\min\{(f_u\wedge g_v)(x),(f_u\wedge g_v)(y)\}$$
$$=\min\{h_p(x),h_p(y)\}$$
$$Now\ h_p(x\alpha s\beta y\gamma t\delta z)=\min\{f_u(x\alpha s\beta y\gamma t\delta z),g_v(x\alpha s\beta y\gamma t\delta z)\}$$
$$\geq\min\{\min\{f_u(s),f_u(t)\},\min\{g_v(s),g_v(t)\}\}$$
$$=\min\{\min\{f_u(s),g_v(s)\},\min\{f_u(t),g_v(t)\}\}$$
$$=\min\{(f_u\wedge g_v)(s),(f_u\wedge g_v)(t)\}$$
$$=\min\{h_p(s),h_p(t)\}$$

Hence h_p is a FITΓI of N. Therefore (h,W,Γ) is a FSITΓI over N.

Def 3.18: Let (f,U,Γ) be a FSTΓ-SR over a TΓ-SR N. A non null FS set (g,V,Γ) over N is called *FSTΓI of* (f,U,Γ) if i) (g,V,Γ) is a FS subset ii) (g,V,Γ) is a FSTΓI over N.

Th 3.19: [5] Every FTΓI of a TΓ-SR is a FITΓI of N.

Corollary 3.20: Let N be a TΓ-SR and (f,U,Γ) be a FSTΓI over a TΓ-SR N then (f,U,Γ) is a FSITΓI over N.

Definition 3.21: Let *f* be a FSS of N and $a \in \left[0,1-\sup\left\{f(x)/x \in N\right\}\right]; b \in [0,1]$

The mappings $f_a^T : N \to [0,1],\ f_b^M : N \to [0,1],\ f_{b,a}^{MT} : N \to [0,1]$ are called FT (fuzzy translation), FM (fuzzy multiplication) and FMT (fuzzy magnified translation) of *f* respectively if $f_a^T(n) = f(n)+a,\ n_b^M(n) = bf(n),\ f_{b,a}^{MT}(n) = bf(n)+a \ \ \forall n \in N.$

Th 3.22: A FSS *f* is an interior $t\Gamma-ideal$ of a TΓ-SR N if and only if Π_a^T, the FT of *f* is an interior $t\Gamma-ideal$ of TΓ-SR N.

Proof: Suppose *f* is a interior t-ideal of N. $x,y,z,s,t \in N, \alpha,\beta,\gamma,\delta \in \Gamma$

$$\begin{aligned}\pi_a^T(x\alpha s\beta y\gamma t\delta z) &= \pi(x\alpha s\beta y\gamma t\delta z)+a \\ &\geq \min\{\pi(s),\pi(t)\}+a \\ &= \min\{\pi(s)+a,\pi(t)+a\} \\ &= \min\{\pi_a^T(s),\pi_a^T(t)\}\end{aligned}$$

Therefore, Π_a^T is a FITΓI of N.

Def 3.23: Let *f* be fuzzy multiplication interior t-ideal of a TΓ-SR N. Then (f,U,Γ) is said to be FMSITΓI (fuzzy multiplication soft interior t-ideal) over N: if f_a is multiplication fuzzy interior t-ideal of ternary Γ-Semiring over N for all $a \in U\ where U = \{a/a \in [0,1]\}.$

Th 3.24: Let N be a TΓ-SR. Then *f* is a interior t-ideal of N if and only if f_b^M, the fuzzy multiplication of *f* is a FIT**Γ**I of a T-ΓSR N.

Proof: Let *f* is a interior t-ideal of N, $p,q,r,s,t \in N, \alpha,\beta,\gamma,\delta \in \Gamma$

$$\begin{aligned}f_b^M(p+q) = bf(p+q) &\geq b\min\{f(p),f(q)\} \\ &= \min\{b\,f(p),b\,f(q)\} \\ &= \min\{f_b^M(p),f_b^M(q)\}\end{aligned}$$

$$\begin{aligned}f_b^M(p\alpha s\beta q\gamma t\delta r) &= b\,f(p\alpha s\beta q\gamma t\delta r) \\ &\geq b\min\{f(s),f(t)\} \\ &= \min\{b\,f(s),b\,f(t)\} \\ &= \min\{f_b^M(s),f_b^M(t)\}\end{aligned}$$

Therefore, f_b^M is a FITΓI of N.

Corollary 3.25: Let *f* be FMITΓI of a TΓ-SR N. Then (f_t,U,Γ) is a FMSITΓI over N. Where $U = \{a/a \in [0,1]\}.$

Th 3.26: If *f* is a FITΓI of a TΓ-SR N. Then (f_t,U,Γ) is a soft interior t-ideal of over N. Where $U = \{a/a \in [0,1]\}.$ and f_t is a non empty level subset of *f*.

Proof: Let N be a ternary Γ-Semiring. Suppose that $f_1 \neq \phi, a \in [0,1]$, $p,q,r \in f_t, \alpha,\beta,\gamma,\delta \in \Gamma$ Then

$f(p) \geq t, f(q) \geq t, f(r) \geq t. \Rightarrow f(p+q) \geq \min\{f(p), f(q)\} \geq \min\{t,t\} = t.$

$\Rightarrow (p+q) \in f_t.$

Also $f(p\alpha q\beta r) \geq \min\{f(p), f(q), f(r)\} \geq \min\{t,t,t\} = t \Rightarrow p\alpha q\beta r \in f_t$.

$Let\, u,v \in N \Rightarrow f(p\alpha u\beta q\delta v\gamma r) \geq \min\{f(u), f(v)\} \geq \min\{t,t\} \geq t \Rightarrow p\alpha u\beta q\delta v\gamma r \in f_t$

Therefore f_t is a interior t-ideal of N.

Th 3.27: If *f* is a fuzzy interior t-ideal of a simple TΓ-SR N then *f* is a constant function.

Proof: Let *f* be a FITΓI of a simple TΓ-SR N and $x \in N$, $Let\, N\Gamma x\Gamma N = \{t \in N / t \leq y, y \in N\Gamma x\Gamma N\}$. Then $N\Gamma x\Gamma N$ is a TΓ-ideal of N. Therefore $N = N\Gamma x\Gamma N.$ Suppose $n \in N \Rightarrow n \in N\Gamma x\Gamma N$ $\Rightarrow n = c\alpha x\beta d, c,d \in N, \alpha,\beta \in \Gamma$ $\Rightarrow f(n) = f(c\alpha x\beta d) \geq f(x)$. lly, we can prove $f(x) \geq f(n). \therefore f(x) = f(n).$

Corollary 3.28: Let *f* be FMITΓI of a simple TΓ-SR N. Then (f_t, U, Γ) is a FS constant function over N. where $U = \{a / a \in [0,1]\}.$

Th 3.29: 59. Let N be a TΓ-SR. Then *f* is a FIT**Γ**I of a TΓ-SR N if and only if $f_{b,a}^{MT}$, FMT of *f* is a FIT**Γ**I of N.

Proof: Suppose *f* is a FIT**Γ**I of N

$\Leftrightarrow$ f_b^M is a FIT**Γ**I of TΓ-SR N by theorem 3.24.

$\Leftrightarrow$ $f_{b,a}^{MT}$ is a FIT**Γ**I of N.

4. CONCLUSION

Conclusion: In this paper we introduce notation of fuzzy soft ternary Γ-Semiring, Fuzzy soft TΓ-ideal, fuzzy soft interior TΓ-ideal over ternary Γ-Semiring and some algebraic properties were studied.

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