A NOVEL STUDY ON CROWD DYNAMICS

M.REVATHY
Research Scholar
Dept. of PG Research in Mathematics
Bharath Institute of Higher Education And Research
Chennai.

KAVITHA
Asst. Professor
Dept. of PG Research in Mathematics
Bharath Institute of Higher Education And Research
Chennai.

1. Introduction

Crowd dynamics can be defined as the study of the how and where crowds form and move above the critical density of more than one person per square metre. At this density there is the potential for overcrowding and personal injury. We examine different approaches used to define crowd safety, in particular the ***Green Guide.*** which defines the criteria for sporting grounds, the ***Purple Guide*** which defines the criteria for pop concerts and similar events and the ***Primrose Guide*** which defines the criteria for existing places of entertainment. We examine the relationships between crowd flow and density, specifically the work of Fruin [6] who has been instrumental in setting the standards for pedestrian planning around the world. In addition to the existing body of literature and guides we perform extensive field studies to examine the nature of crowd dynamics with respect to local geometry, for example, building entrances, turnstiles and corridors.

2.An Overview of Models for the Simulation of Pedestrian Dynamics

The simulation of pedestrian movement has being explored in a variety of ways. In this chapter the aim will be to provide an outline of the most prominent of these models along with an exposition of the governing equations of each scheme. In general these models describe the forces each pedestrian feels; treating each pedestrian as a particle in a larger system and using Newton's Second Law to evaluate position, velocity and acceleration. A numerical solver then can be used over a discrete timestep providing a velocity and position update. One of the key differences between pedestrian traffic models and other roadway-based traffic systems is that pedestrian locations are not restricted to a single dimension. Whilst many of the models are initially based on vehicle traffic systems, these are only one dimensional models and unlike pedestrian movement subject to a number of laws and restrictions governing traffic. Pedestrian movement is inherently more changeable than that of vehicles, as unlike vehicle flow which is controlled by well defined lane markings with lane change and passing opportunities restricted, there are no such restrictions on pedestrian walkways. Pedestrian interaction is also markedly different to that of cars since safety concerns are much less, clearly pedestrians can actually touch each other without incident, which is certainly not true of moving vehicles. Also pedestrians often move in pair or clusters, such as couples or family groups, whilst such attractive influences are rare in vehicle traffic. This leads to interactions between people that have to be considered in any model, examples of which are bumping into each other, exchange of places or bypass when pedestrian density is high as opposed to sidestepping which would be analogous to the behaviour of a car. The velocity and acceleration characteristics of pedestrians are also very different, with each pedestrian having their own desired and maximum speeds. They are also able to accelerate to full speed from standstill almost immediately and can change speed more rapidly allowing them to take advantage of gaps in traffic when they arise.

3. Queuing Systems

Queuing theory, which is generally considered to be a branch of operations research, describes pedestrian flows in terms of probability functions. The pedestrian will arrive at a given node, which represents a server, with a certain probability. They will then spend a certain amount of time being served, at a shop till for example, and then continue on to their next destination, leaving the queue. A queuing system is comprised of three elements, the pedestrian's arrival in the queue, the service mechanism and the service discipline. A queuing discipline determines the manner in which the exchange handles calls from customers. Examples are

- *First In, First Out* – This principle states that customers are served one at a time and that the customer that has been waiting longest is served first
- *Last In First Out* – This principle also serves customers one at a time, however the customer with the shortest waiting time will be served first
- *Processor Sharing* – Customers are served equally. Network capacity is shared between customers and they all effectively experience the same delay

Queuing is handled by control processes within exchanges, which can be modelled using state equations. Queuing systems use Markov Chains which model the system in each state where Incoming traffic to these systems is modelled via a Poisson distribution. The stochastic process in a queuing system is the population of a particular room.

The first model to be discussed was formulated by S. J. Yuhaski, Jr and J. Macgregor Smith[7] which develops a state dependent queuing model for the congestion effects of movement through circulation systems of a building. Circulation systems are, in this case, the pathways of movement such as corridors, stairways and ramps. The problem they describe is that of crowded pedestrian flows in confined spaces and the paper describes the following as "crucial aspects to movement Systems"

These characteristics are the basis for the construction of their model and their aim to "best capture the congestion effects". The first step is to generate a representation for the facility; this is its floor plan and so the queuing network is the Dual Graph of G□□. There are then two distinct types of spatial entity which must be distinguished, that is the activity network and the circulation network.

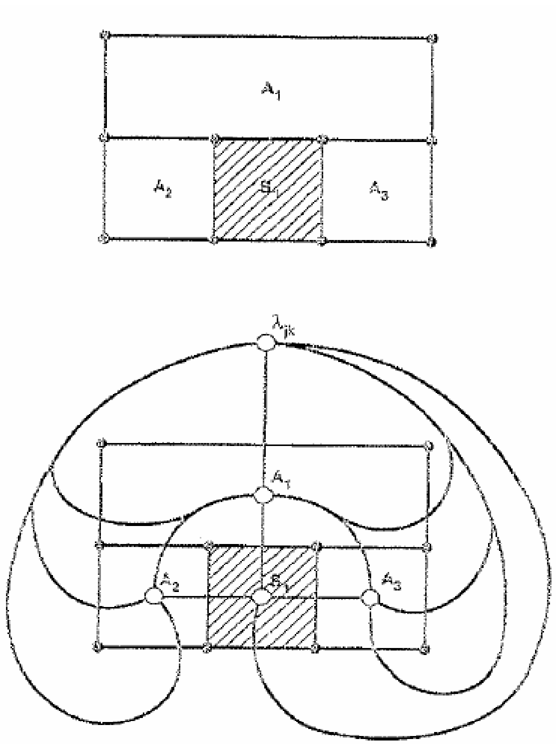

Figure 1: Planar Graph $G'(V',E')$ and dual graph $G(V,E)$

One feature of modelling pedestrian facilities is that transitions within a network are not virtually instantaneous as in other systems (telephone networks for example) and so the set S is required.

4.Friction Based Traffic Flow Model

Traffic flow problem is a scalar, time –varying, non-linear, hyperbolic partial differential equation where traffic density is conserved. It is presented as two-equation model, the first equation is conservation of mass and the second equation is velocity equation Friction on the surface of the road, strike, VIP arrival, snow fall, rain, cyclone, Tsunami, earthquake etc are factors affecting the speed of the vehicle and form a cause for formation of crowd traffic.
According the velocity changes,

the convective derivative in the general form is given by

$$\frac{D\varphi}{Dt} = \frac{\partial\varphi}{\partial t} + (\vec{v}.\nabla)\varphi$$

The first PDE equation is given by,

$$\rho_t + (\rho v)_x = 0 \qquad (5.1)$$

In the present case, the second equation is :

PDE is $$\left(v + P(\rho)\right)_t + v\left(v + P(\rho)\right)_x = 0 \qquad (5.2)$$

The above equation is known as "traffic motion dynamics" . This model caries the anisotropic property, and it is given by

$$P(\rho) = c_0^2\, \rho^r \ , \qquad (5.3)$$

where $\gamma > 0$ and $c_0 = 1$.

We will put the second equation in conservation form and find the system eigen values.
Consider

$$\left(v + P(\rho)\right)_t + v\left(v + P(\rho)\right)_x = 0$$

Multiplying this equation by ρ we get ,

$$\rho[\left(v + P(\rho)\right)_t + v\left(v + P(\rho)\right)_x] = 0$$
$$\Rightarrow \rho\left(v + P(\rho)\right)_t + \rho v\left(v + P(\rho)\right)_x = 0 \qquad (5.4)$$

Using the product rule,

$$\left(\rho\left(v + P(\rho)\right)\right)_t = \rho_t\ \left(v + P(\rho)\right) + \rho\left(v + P(\rho)\right)_t$$

and

$$\rho v\left(v + P(\rho)\right)_x = (\rho v)_{x\,(v+P(\rho))} + (\rho v)\ (v + P(\rho))_x$$

Substituting the above value in (5.2),

$$(\rho\left(v + P(\rho)\right)_t + \rho_t(\ v + P(\rho) + (\rho v)_x\ (\ v + P(\rho) + \ (\rho v)\left(v + P(\rho)\right)_x = 0 \quad (5.5)$$

Now using the conservation law for ρ_t, the above equation can be simplified as

$$\rho\big(v + P(\rho)\big)_t + \rho v\,\big(v + P(\rho)\big)_x = 0 \qquad (5.6)$$

Then above equation is the conservation form of equation (5.4) and our conserved variables are ρ and $\rho\big(v + P(\rho)\big)$.

Let $X = \rho\big(v + P(\rho)\big)$

Consider $\rho_t + (\rho v)_x = 0$.

$$\text{As } X = \rho v + \rho\, P(\rho) \qquad (5.7)$$

$X - \rho P(\rho) = \rho v$

$\therefore \rho_t + (X - \rho P(\rho))_x = 0$

$(\rho\big(v + P(\rho)\big))_t + (\rho v\,(v + P(\rho)))_x = 0$

$$X_t + \rho v\,(v + P(\rho)))_x = 0 \qquad (5.8)$$

Consider $\rho v\,(v + P(\rho)))_x$

$\rho v = X - \rho P(\rho)$ *from equation (5.7)*

$\Rightarrow X = \rho[v + P(\rho)]$

$\Rightarrow \frac{X}{\rho} = v + P(\rho)$

$\therefore [X - \rho P(\rho)]\,[V + P(\rho)] = 0$

$[X - \rho P(\rho)]\,\frac{X}{\rho} = 0$.

$\frac{X^2}{\rho} - XP(\rho) = 0$

$\therefore$ Using this Equation (5.8) becomes,

$$X_t + \frac{X^2}{\rho} - XP(\rho)_x = 0$$

$$\therefore \begin{cases} \rho_t + (X - \rho P(\rho))_x = 0 \\ X_t + \left(\frac{X^2}{\rho} - XP\,(\rho)\right)_x = 0 \end{cases} \qquad (5.9)$$

Since this falls in the vector form of $Q_t + F(Q)_x = s$, states and the flux are given by

$$Q = \begin{bmatrix} \rho \\ X \end{bmatrix},\; F(Q) = \begin{bmatrix} X - \rho P(\rho) \\ \frac{X^2}{\rho} - XP(\rho) \end{bmatrix}.$$

For the quasi-linear form $Q_t + A(Q)Q_x = 0$, the Jacobin is given by

$$A(Q) = \frac{\partial F}{\partial Q} = \begin{bmatrix} -(\gamma+1)P(\rho) & 1 \\ -\left(\frac{X^2}{\rho^2} + \frac{\gamma X P(\rho)}{\rho}\right) & \left(\frac{2X}{\rho} - P(\rho)\right) \end{bmatrix}$$

Finally, solving for the eigen values from $|A(Q) - \lambda I| = 0$,
we find two distinct and real eigen values,
Consider,

$$A = \frac{\partial F}{\partial Q} = \begin{bmatrix} -(\gamma+1)P(\rho) & 1 \\ -\left(\frac{X^2}{\rho^2} + \frac{\gamma X P(\rho)}{\rho}\right) & \left(\frac{2X}{\rho} - P(\rho)\right) \end{bmatrix}$$

$$|A - \lambda I| = \begin{bmatrix} -(\gamma+1)P(\rho) - \lambda & 1 \\ -\left(\frac{X^2}{\rho^2} + \frac{\gamma X P(\rho)}{\rho}\right) & \left(\frac{2X}{\rho} - P(\rho)\right) - \lambda \end{bmatrix} = 0$$

$$\Rightarrow [-(\gamma+1)P(\rho) - \lambda]\left[\frac{2X}{\rho} - P(\rho) - \lambda\right] + \left[\frac{X^2}{\rho^2} + \frac{\gamma X P(\rho)}{\rho}\right] = 0$$

$$\Rightarrow \frac{-(\gamma+1)P(\rho)2X}{\rho} + (\gamma+1)(P(\rho))^2 + \lambda(\gamma+1)P(\rho) - \frac{2X\lambda}{\rho} + \lambda P(\rho) + \lambda^2 + \frac{X^2}{\rho^2} + \frac{\gamma X\, P(\rho)}{\rho} = 0$$

$$\Rightarrow \lambda^2 + \lambda\ [(\gamma+1)\ P\ (\rho)\ - \frac{2X}{\rho} + P(\rho)] + \frac{\gamma X P\,(\rho)}{\rho} - \frac{(\gamma+1)P(\rho)2X}{\rho} + (\gamma+\ 1)(P(\rho))^2 + \frac{X^2}{\rho^2} = 0$$

$$\Rightarrow \lambda^2 + \ \lambda\left(\gamma\, P(\rho) - \frac{2X}{\rho} + 2P(\rho)\right) - \frac{\gamma X P(\rho) - \gamma P(\rho)2X - P(\rho)2X}{\rho} + (\gamma+1)(P(\rho))^2 + \frac{X^2}{\rho^2} = 0$$

$$\Rightarrow \lambda^2 + \ \lambda\left(\gamma\, P(\rho) - \frac{2X}{\rho} + 2P(\rho)\right) - \frac{\gamma X P(\rho) - P(\rho)2X}{\rho} + (\gamma+1)(P(\rho))^2 + \frac{X^2}{\rho^2} = 0 \quad (5.10)$$

Consider, $\gamma\, P(\rho) - \frac{2X}{\rho} + 2P(\rho)$

Subtituting $X = \rho[v + P(\rho)]$, *in the above expression , we get*

$$\gamma\, P(\rho) - \frac{2\rho[v + P(\rho)]}{\rho} + 2P(\rho)$$

$$= \ \gamma\, P(\rho) - 2V - 2P(\rho) + 2P(\rho)$$

$$= -2V + \gamma\, P(\rho)$$

$$= -(2V - \gamma\, P(\rho))$$

Now Consider,

$$-\frac{\gamma X P(\rho)-P(\rho)2X}{\rho}+(\gamma+1)(P(\rho))^2+\frac{X^2}{\rho^2}$$

$$=-\frac{\gamma X P(\rho)-P(\rho)2\rho(V+P(\rho))}{\rho}+(\gamma+1)(P(\rho))^2+\frac{(\rho(V+P(\rho)))^2}{\rho^2}$$

$$=\frac{-\gamma P(\rho)X}{\rho}-2V\big(P(\rho)\big)-2(P(\rho))^2+\gamma(P(\rho))^2+(P(\rho))^2+(V+P(\rho))^2$$

$$=\frac{-\gamma P(\rho)X}{\rho}-2V\big(P(\rho)\big)-2(P(\rho))^2+\gamma(P(\rho))^2+(P(\rho))^2+V^2+(P(\rho))^2-2VP(\rho)$$

$$=\frac{-\gamma P(\rho)\rho(V+P(\rho))}{\rho}+V^2$$

$$=-\gamma P(\rho)V-\gamma(P(\rho))^2+V^2+\gamma(P(\rho))^2$$

$$=V(V-\gamma P(\rho))$$

$\therefore$ Equation (5.10) becomes,

$$\lambda^2+\lambda\big[-\big(2V-\gamma P(\rho)\big)\big]+V(V-\gamma P(\rho))=0$$

$$\lambda^2-2V\lambda-\lambda\gamma P(\rho)+V^2-V\gamma P(\rho)=0$$

$$\lambda^2-V\lambda-V\lambda-\lambda\gamma P(\rho)+V^2-V\ =0$$

$$\Rightarrow\lambda=\frac{(2V-\gamma P(\rho))\pm\sqrt{(2V-\gamma P(\rho)\,)^2-4(1)(V^2-V\gamma P(\rho))}}{2(1)}$$

$$\lambda=\frac{(2V-\gamma P(\rho))\pm\sqrt{4V^2+\gamma^2(P(\rho))^2-2(2V)\gamma P(\rho)-4V^2+4V\gamma P(\rho)}}{2}$$

$$\lambda=\frac{(2V-\gamma P(\rho))\pm\sqrt{\gamma^2(P(\rho))^2}}{2}$$

$$\lambda=\frac{(2V-\gamma P(\rho))\pm\gamma P(\rho)}{2}$$

$$\lambda=\frac{(2V-\gamma P(\rho))+\gamma P(\rho)}{2}$$

$$\lambda=\frac{2V}{2}$$

$$\lambda=\ V$$

Consider

$$\lambda=\frac{(2\mathrm{V}-\gamma\mathrm{P}(\rho))-\gamma\mathrm{P}(\rho)}{2}$$

$$\lambda=\frac{(2V-2\gamma P(\rho))}{2}$$

$$\lambda=V-2\gamma P(\rho)$$

$$\lambda_1=v_F-\gamma P(\rho)\ ,\ \ \lambda_2=v_{\ F}.$$

Introducing frictional force as a factor reducing the speed of the car, we get,

$\lambda_1 = v_F - \gamma P(\rho)$, $\lambda_2 = v_F$.

where v_F = changed speed after friction is introduced.

5.3 Types of Friction

There are two types of friction .

- Tyre friction
- Road surface friction

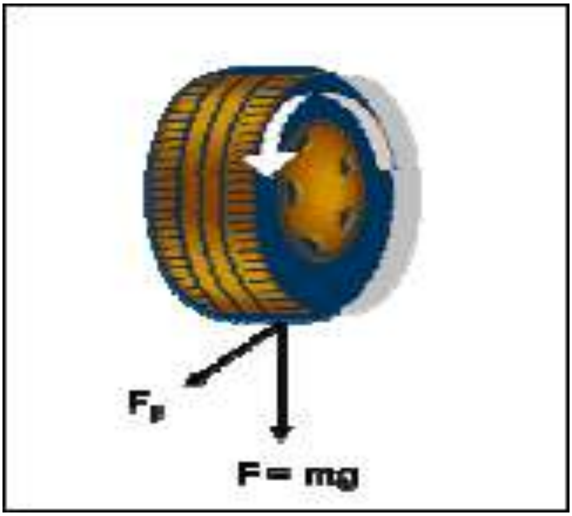

Fig. 2. Friction force Fμ and the normal force F = mg, where m is the mass supported by the wheel and g is the acceleration of gravity g = 9.81 m/s.

Friction force will induce an acceleration $\boldsymbol{a}$ of the mass m supported by the tyre, thus and $F_\mu = ma$. Since the gravity will induce a normal force $F = mg$, where g = 9.81 m/s², we finally can conclude that $\mu = \boldsymbol{a}/g$

Thus (1) friction should not depend on the mass of the car .

(2) friction can be measured directly by measuring the acceleration while braking or speeding up so that tires slip or get at least near to slipping. If not all the mass of the car is supported by

braking or speeding wheels the above equation needs to be adjusted accordingly.

Friction coefficient represents the maximum attainable acceleration, excluding collision, in fractions of acceleration of gravity and is a very good measure of slipperiness. Not all tyres are the same or pavement surfaces.

Nevertheless, typical friction coefficients of a rubber tyre on a dry surface are about 0.80 ± 0.10 and reduction of friction down to 0.20 level with hard ice can happen even with studded tires. One of those other forces could be air or wind drag, but they cause a tolerable effect at low car speeds less than 100 km/h or wind speeds less than 20 m/s.

There is also another interesting result in close proximity to the equation of friction and acceleration. If the car is running at an initial speed v_i, then the kinetic energy of the car is E_k = m vi^2 / 2. This energy can be consumed by lock braking. Thus the kinetic energy must equal to the work done by the friction force in this case and we get

$$E_k = F_\mu l$$

where ℓ is the stopping distance from the initial speed vi to zero speed while lock braking. This equality gives us

$$l = {v_i}^2/2\mu g \qquad \text{or} \quad \mu = {v_i}^2/2gl$$

so that we can measure friction coefficient by observing the initial speed and measuring the braking distance.

In Fig. 2 there is shown the braking distance as a function of initial speed with a given coefficient of friction within μ = 0.15 – 0.80. This relation tells us important facts about driving in winter conditions. Let us say we are driving at a speed of 100 km/h on a dry road. Then we see that the stopping distance is about 50 m. If there is a thin layer of hard ice on the surface, friction coefficient may reduce close to about 0.20. In this case the stopping distance would increase to about 200 m, which is four times longer than 50 m on a dry road. It would take over 14 seconds to stop the car without collision, which is an astonishingly long time, not easy to believe without an experience.

6. Conclusion

The highest friction levels on any particular road are likely to be achieved when tyres are new. When the road is dry, the new tyre

polymer can grip the surface and when it is wet, not only is the new polymer better able to grip the relatively dry part of the contact patch, the full tread depth is available to assist in the bulk removal of water and offset any shortcomings in the road surface texture.
However, as the tyre wears and ages, it is likely that available friction will be reduced. The tyre tread compounds will gradually harden, reducing the maximum grip that can be achieved, but the greatest influence is in the reduced tread depth as the tyre wears.

7. Reference

[1] D. Helbing et al. Self-Organized Pedestrian Crowd Dynamics: Experiments, Simulations, and Design Solutions. Transportation Science, 39(1):1-24, February 2005.
[2] D. Helbing and P. Molnár. Social force Model for Pedestrian Dynamics. Physical Review E, 51(5):4282-4286, May 1995
[3] V. J. Blue and J. L. Adler. Bi-directional Emergent Fundamental Pedestrian Flows from Cellular Automata Microsimulation. Proceedings of the 14thInternational Symposium of Transportation and Traffic Theory, 1999.
[4] L. F. Henderson. The Statistics of Crowd Fluids, Nature, 229:381-383, February 1971.
[5] S. Hoogendoorn and P. H. L. Bovy. Gas-Kinetic Modeling and Simulation of Pedestrian Flows. Transportation Research Record, 1710:28-36, 2000.
[6] S. Okazaki and S. Matsushita. A Study of Simulation Model for Pedestrian Movement with Evacuation and Queuing. Proceedings of the International Conference on Engineering for Crowd Safety, pages 271-280, 1993.
[7] S. J. Yuhaski, Jr and J. Macgregor Smith. Modeling Circulation Systems in Buildings Using State Dependent Queuing Models. Queueing Systems, pages 319-338,1989.
[8] G. G. Løvas. Modeling and Simulation of Pedestrian Traffic flow. Transportation Research B, 28(6):429-443, 1994.
[9] V. J. Blue and J. L. Adler. Cellular Automata Microsimulation of Bi-Directional Pedestrian Flows, Transportation Research Record, 1678:135-141, 2000.
[10] J. Dijkstra, J. Jessurun and H. Timmermans. A Multi-Agent Cellular Automata Model of Pedestrian Movement. Pedestrian and Evacuation Dynamics, pages 173-181,2001.

[11] D. Helbing and P. Molnár. Self-Organization Phenomena In Pedestrian Crowds. Self-Organization of Complex Structures: From Individual to Collective Dynamics, pages 569-577, 1997.
[12] J. Dzubiella and H. Löwen. Pattern Formation in Driven Colloidal Mixtures: Tilted Driving Forces and Re-Entrant Crystal Freezing. J. Physics: Condensed Matter 14:9383- 9395, 2002.
[13] L. Verlet. Computer “Experiments” on Classical Fluids. I. Thermodynamical Properties of Lennard-Jones Molecules. Physical Review, 159(1):98-103.
[14] B. D. Ripley. Stochastic Simulation John Wiley & Sons, Inc, 1987.
[15] P. Amorim, R. M. Colombo and A. Teixeira, On the numerical integration of scalar conservation laws, preprint, 2013.
[16] A. Aw and M. Rascle, Resurrection of second order" models of traffic ow, SIAM J. Appl.Math., 60 (2000), 916-938 (electronic).
[17] N. Bellomo and C. Dogbffie, On the modelling crowd dynamics from scaling to hyperbolic macroscopic models, Math. Models Methods Appl. Sci., 18 (2008), 1317-1345.
[18] N. Bellomo and C. Dogbffie, On the modeling of traffic and crowds: A survey of models, speculations, and perspectives, SIAM Review, 53 (2011), 409-463.
[19] R. M. Colombo, M. Garavello and M. Lecureux-Mercier, A class of non-local models for pedestrian traffic, Mathematical Models and Methods in the Applied Sciences, 22 (2012), 1150023, 34 p.