ON A SET OF THREE DIOPHANTINE EQUATIONS

$x+y=2a^2, 2x+y=5a^2-b^2, x+2y=5c^3$

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ABSTRACT

The system of triple equations with five unknowns represented by $x+y=2a^2, 2x+y=5a^2-b^2, x+2y=5c^3$ is analyzed for its non-zero distinct integral solutions. Different sets of solutions are presented.

Keywords: System of triple equations , triple equations with five unknowns, integer solutions.

1. INTRODUCTION

In [1], an attempt has been made to obtain pairs of non-zero distinct integers x, y such that, in each pair

i. $x+y=a^2, 2x+y=b^2, x+2y=a^3$
ii. $x+y=a^2, 2x+y=b^2, x+2y=c^3$

[2] illustrates the analysis of obtaining different sets of distinct integer solutions to two systems of triple equations with five unknowns given by

i. $x+y=a^2, 2x+y=b^2, x+2y=3c^2$
ii. $x+y=a^2, 2x+y=b^2, x+2y=2c^2$ respectively.

In [3], the system of three equations $x+y=a^2, 2x+y=b^2, x+2y=a^2-c^2$ has been studied for its non-zero distinct integer solutions.

In [4-6], the following three systems of Triple Equations are studied :

i. $x+y=a^2, 2x+y=a^2+3b^2, x+2y=a^2+c^2$
ii. $x+y=a^2, 2x+y=a^2+b^2, x+2y=a^2+5c^2$
iii. $x+y=2a^2, 2x+y=5a^2+b^2, x+2y=c^3$

This communication exhibits different sets of non-zero distinct integer solutions for the system of triple equations with five unknowns given by $x+y=2a^2, 2x+y=5a^2-b^2, x+2y=5c^3$.

2. METHOD OF ANALYSIS

The system of triple equations with five unknowns to be solved for its distinct non-zero integral solutions are

$$x + y = 2a^2 \tag{1}$$

$$2x + y = 5a^2 - b^2 \tag{2}$$

$$x + 2y = 5c^3 \tag{3}$$

Eliminating x and y between (1) to (3) , we get

$$a^2 + b^2 = 5c^3 \tag{4}$$

Also, solving (1) and (2) for x and y, one obtains

$$x = 3a^2 - b^2 \tag{5}$$

$$y = b^2 - a^2 \tag{6}$$

Now, solving (4), the values of a, b ,c are obtained. In view of (5) and (6), the values of x and y satisfy (1) to (3) are obtained. Thus, the above values of x, y, a, b and c represent the solutions to the system of equations (1) to (3). The above process is illustrated as follows:

Method 1:

Let

$$c = \alpha^2 + \beta^2 \tag{7}$$

where α and β are non-zero integers.

Write 5 as

$$5 = (2 + i)(2 - i) \tag{8}$$

Using (7),(8) in (4) and applying the method of factorization, define

$$(a + ib) = (2 + i)[(\alpha + i\beta)]^3 \tag{9}$$

from which we have

$$\left.\begin{aligned} a = a(\alpha,\beta) = 2\alpha^3 - 6\alpha\beta^2 - 3\alpha^2\beta + \beta^3 \\ b = b(\alpha,\beta) = \alpha^3 - 3\alpha\beta^2 + 6\alpha^2\beta - 2\beta^3 \end{aligned}\right\} \tag{10}$$

Substituting (10) in (5) and (6), the corresponding values of x and y are obtained.

A few numerical examples are given in the Table 1 below:

Table 1 Numerical values

α	β	a	b	c	x	y
1	2	-20	-15	5	975	-175
2	3	-101	-28	13	29819	-9417
1	3	-34	-62	10	-376	2688
2	4	-160	-120	20	62400	-11200

Method 2:

Write 5 as

$$5=(1+2i)(1-2i) \tag{11}$$

Using (11), (7) in (4) and applying the method of factorization, define

$$(a+ib)=(1+2i)[(\alpha+i\beta)]^3 \tag{12}$$

Equating real and imaginary parts, we have

$$\left.\begin{aligned} a=a(\alpha,\beta)=\alpha^3-3\alpha\beta^2-6\alpha^2\beta+2\beta^3 \\ b=b(\alpha,\beta)=2\alpha^3-6\alpha\beta^2+3\alpha^2\beta-\beta^3 \end{aligned}\right\} \tag{13}$$

Substituting (13) in (5) and (6), the corresponding values of x and y are obtained.

A few numerical examples are given in the Table 2 below:

Table 2 Numerical values

α	β	a	b	c	x	y
1	2	-7	-24	5	-429	527
2	3	-64	-83	13	5399	2793
1	3	10	-70	10	-4600	4800
2	4	-56	-192	20	-27456	33728

Method 3:

Substituting

$$a=(5k-3)b \tag{14}$$

in (4), it gives

$$(5k^2-6k+2)b^2=c^3 \tag{15}$$

Choose b and c such that

$$b=\left(5k^{2}-6k+2\right)u^{3} \tag{16}$$

$$c=\left(5k^{2}-6k+2\right)u^{2} \tag{17}$$

Note that (15) is satisfied.

Using (16) in (14), we have

$$a=(5k-3)\left(5k^{2}-6k+2\right)u^{3}$$

It is observed that the values of a, b and c represented by

$$a=(5k-3)\left(5k^{2}-6k+2\right)u^{3}$$

$$b=\left(5k^{2}-6k+2\right)u^{3}$$

$$c=\left(5k^{2}-6k+2\right)u^{2}$$

satisfy (4). In view of (5) and (6) the values of x and y satisfying (1) to (3) are obtained.

$$x=u^{6}\left(5k^{2}-6k+2\right)^{2}\left(75k^{2}-90k+26\right)$$
$$y=-u^{6}\left(5k^{2}-6k+2\right)^{2}\left(25k^{2}-30k+8\right)$$

A few numerical examples are given in the Table 3 below:

Table 3 Numerical values

u	k	a	b	c	x	y
2	1	2	8	4	11	-3
2	2	70	80	40	14600	-4800
2	3	348	232	116	362471	-120263
2	4	986	464	232	2913224	-968832

Method 4:

Write (4) as

$$a^{2}+b^{2}=5c^{3}*1 \tag{18}$$

Write 1 as

$$1=\frac{(3+4i)(3-4i)}{25} \tag{19}$$

Using (7), (8), (19) in (18) and applying the method of factorization, define

$$(a+ib)=(2+i)[(\alpha+i\beta)]^{3}\frac{(3+4i)}{5} \tag{20}$$

from which we have

$$\left.\begin{aligned} a = a(\alpha,\beta) = \frac{1}{5}\left[2\alpha^3 - 6\alpha\beta^2 - 33\alpha^2\beta + 11\beta^3\right] \\ b = b(\alpha,\beta) = \frac{1}{5}\left[11\alpha^3 - 33\alpha\beta^2 + 6\alpha^2\beta - 2\beta^3\right] \end{aligned}\right\} \quad (21)$$

Since our interest is on finding integer solutions, replacing, α by $5A$ and β by $5B$ in (7) and (21), the corresponding integer solutions to a, b and c are given by

$$a(A,B) = 50A^3 - 150AB^2 - 825A^2B + 275B^3$$
$$b(A,B) = 275A^3 - 825AB^2 + 150A^2B - 50B^3$$
$$c(A,B) = 25(A^2 + B^2)$$

In view of (5) and (6), the values of x and y are obtained.

A few numerical examples are given in the Table 4 below:

Table 4 Numerical values

α	β	A	B	a	b	c	x	y
5	15	1	3	3650	-8050	250	-24835000	51480000
10	15	2	3	-4775	-12200	325	-80438125	126039375
15	10	3	2	-13100	-175	325	514799375	-171579375
10	30	2	6	29200	-64400	1000	-1589440000	3294720000

Method 5:

Using (7), (11), (19) in (18) and applying the method of factorization, define

$$(a + ib) = (1 + 2i)[(\alpha + i\beta)]^3 \frac{(3 + 4i)}{5} \quad (22)$$

Equating real and imaginary parts, we have

$$\left.\begin{aligned} a = a(\alpha,\beta) = -\alpha^3 + 3\alpha\beta^2 - 6\alpha^2\beta + 2\beta^3 \\ b = b(\alpha,\beta) = 2\alpha^3 - 6\alpha\beta^2 - 3\alpha^2\beta + \beta^3 \end{aligned}\right\} \quad (23)$$

Substituting (23) in (5) and (6), the corresponding values of x and y are obtained.

A few numerical examples are given in the Table 5 below:

Table 5 Numerical values

α	β	a	b	c	x	y
1	2	15	-20	5	275	175
2	3	28	-101	13	-7849	9417
1	3	62	-34	10	10376	-2688
2	4	120	-160	20	17600	11200

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