Strong Anti S -Fuzzy Bi-ideals of BCK – Algebras

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Abstract- **In this paper, the notion of strong anti S-fuzzy bi-ideal of a BCK-Algebra are introduced and studied. We show that a collection of strong S- fuzzy bi-ideal we established strong anti S-fuzzy bi-ideal of a BCK-Algebra. We also fuzzify α is a strong anti S- fuzzy bi-ideal in X iff the α^c is a strong S- fuzzy bi-ideal in X.The concept of fuzzify strong anti S- fuzzy bi-ideal and strong anti S- fuzzy subalgebra of a BCK-Algebra are introduced some of their properties are investigated.**

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I. Introduction

After the concept of fuzzy sets by Zadeh. These ideas have been applied to other algebraic structures such as semi groups, groups,rings,modules,vector spaces and topologies. In 1990,Biswas defined the concept of anti-fuzzy subgroup of group and how recently Hong and Jun, modifying Biswas idea, applied the idea of BCK-Algebras.

In this paper we introduce Strong S- fuzzy bi-ideal and Strong anti S-fuzzy bi-ideal of a BCK-Algebra. We also fuzzify α is a Strong anti S- fuzzy bi-ideal in X iff α^c is a Strong S-fuzzy bi-ideal in X.We suggest the perception of a Strong anti S-fuzzy bi-ideal and Strong Anti- S fuzzy sub- algebra of a BCK-Algebra and explore some related properties.

II. PRELIMINARIES

Definition2.1:[2] An-algebra we mean a non-empty set X with a binary operation $*$ and a constant 0 satisfying the following condition:

(a) $((x* y) * (x* z) * (z* y)) = 0$

(b) $(x* (x* y)) * y) = 0$

(c) $x* x = 0$

(d) $0* x = 0$

(e) $x* y = 0$ and $y* x = 0$ imply that $x = y$ for all $x, y, z \in X$.

Definition2.2:[6] A partial ordering "≤ " on X can be defined by $x \leq y$ iff $x * y = 0$.

Definition2.3:[2] A non-empty subset S of a BCK-algebra X is called a sub -algebra of X if $x * y \in S$ whenever $x, y \in S$.

Definition2.4:[6] A non-empty subset I of a BCK-algebra X is called an ideal of X if

(a) $0 \in I$

(b) $x * y \in I$ and $y \in I$ imply that $x \in I$ for all $x, y \in X$.

Definition2.5:[6] A fuzzy set α in a non-empty set X we mean a function $\alpha : X \to [0,1]$.

Definition2.6:[6] The complement of α, denoted by $\overline{\alpha}$ is the fuzzy set in X given by $\overline{\alpha}(x) = 1- \alpha(x)$ for all $x \in X$.

Definition2.7:[2] A fuzzy set α in a BCK-algebra X is called a fuzzy ideal of X if

(a) $\alpha(0) \geq \alpha(x)$ for all $x \in X$

(b) $\alpha(x) \geq \min \{ \alpha(x * y), \alpha(y) \}$ for all $x, y \in X$.

Definition2.8:[6] A mapping $f: X \to Y$ of BCK-algebras is called a homomorphism if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$

Definition2.9:[1] A family of fuzzy set $\{\alpha_i / i \in \wedge\}$ is a BCK-Algebra X,the union $\bigvee_{i\in\wedge}\alpha_i$ of $\{\alpha_i / i \in \wedge\}$ is defined by $\bigvee_{i\in\wedge}\alpha_i(x) = \sup\{ \alpha_i / i \in \wedge\}$ for each $x \in X$

Definition2.10:[1] A family of fuzzy set $\{\alpha_i / i \in \wedge\}$ is a BCK-Algebra X,the intersection $\bigwedge_{i\in\wedge}\alpha_i$ of $\{\alpha_i / i \in \wedge\}$ is defined by $\bigwedge_{i\in\wedge}\alpha_i(x) = \inf \{\alpha_i / i \in \wedge\}$ for each $x \in X$.

Definition2.11:A fuzzy set α in a BCK-algebra X is called a strong S-fuzzy sub-algebra of X if $\alpha(y * z, s) \geq \min \{\alpha(y, s), \alpha(z, s)\}$ for all $y, z \in X$.

Definition2.12: A fuzzy subset α in a BCK-algebra X is called a Strong S- fuzzy Bi-ideal if it satisfies:

(a) $\alpha(0, s) \geq \alpha(x, s)$

(b) $\alpha(y * z, s) \geq \min\{ (\alpha(z * (y * z)) * x, s), \alpha(x, s)\}$ for all $x, y, z \in X$.

Example 2.13:
Consider a BCK-algebra X = {0,1,2,3} with the following cayley table.

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	0
3	3	2	1	0

Define a fuzzy set $\alpha : X \to [0,1]$ by $\alpha(0) = \alpha(1) = 0.1$, $\alpha(2)=0.2$, $\alpha(3)=0.5$. Then by routine calculation it is easy to verify that α is a strong S- fuzzy bi-ideal of a BCK-Algebra X.

III. STRONG ANTI S-FUZZY BI-IDEAL OF BCK-ALGEBRA

Definition 3.1:A fuzzy subset α in a BCK-Algebra X is called strong anti S -fuzzy bi-ideal
if i) $\alpha(0, s) \leq \alpha(x ,s)$
ii) $\alpha(y * z, s) \leq \max \{ \alpha((z * (y * z)) * x, s), \alpha(x, s)\}$ for all $x, y, z \in X$.

Example 3.2:
Consider a BCK-algebra X = {0,a,b,c} with the following cayley table.

*	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	a	0	0
c	c	b	a	0

Define a fuzzy set α : X →[0,1] by α (0) = α (a) =.0.5, α (b)=0.4, α (c)=0.3. Then by routine calculation it is easy to verify that α is a strong anti S- fuzzy bi-ideal of a BCK-Algebra X.

Theorem 3.3: If $\{\alpha_i / i \in \wedge\}$ is a family of strong anti S-fuzzy bi-ideals of BCK- Algebra Y then $\bigvee_{i\in\wedge}\alpha_i$ is a strong anti S-fuzzy bi-ideal of a BCK-Algebra Y.

Proof. Let $\{\alpha_i / i \in \wedge\}$ be a family of strong anti S-fuzzy bi-ideals of Y.

Let x, y, z ∈ Y.

Then we have,

$$\bigvee_{i\in\wedge}\alpha_i\,(y * z, s) = \sup\{\alpha_i\,(y * z, s) / i \in \wedge\}$$

$$\leq \sup\{\max\{\alpha_i\,((z* (y* z)) * x, s), \alpha_i\,(x, s) / i \in \wedge\}$$

$$= \max\{\sup\{\alpha_i\,((z* (y* z)) * x, s) / i \in \wedge\}, \sup\{\alpha_i\,(x, s) / i \in \wedge\}\}$$
$$= \max\{(\bigvee_{i\in\wedge}\alpha_i)((z* (y* z)) * x, s), (\bigvee_{i\in\wedge}\alpha_i)(x, s)\}$$

Therefore $\bigvee_{i\in\wedge}\alpha_i(y * z, s) \leq \max\{(\bigvee_{i\in\wedge}\alpha_i)((z* (y* z)) * x, s), (\bigvee_{i\in\wedge}\alpha_i)(x, s)\}$

Hence $\bigvee_{i\in\wedge}\alpha_i$ is a strong anti S-fuzzy bi-ideal of Y.

Theorem 3.4:.If $\{\alpha_i / i \in \wedge\}$ is a family of strong anti S-fuzzy bi-ideals of BCK- AlgebraY then $\bigwedge_{i\in\wedge}\alpha_i$ is a strong anti S-fuzzy bi-ideal of a BCK-Algebra Y.

Proof. Let $\{\alpha_i / i \in \wedge\}$ be a family of strong anti s-fuzzy bi-ideals of Y.

Let x, y, z ∈ Y.

Then we have,

$$\bigwedge_{i\in\wedge}\alpha_i\,(y * z, s) = \inf\{\alpha_i\,(y * z, s) / i \in \wedge\}$$

$$\leq \inf\{\max\{\alpha_i\,((z* (y* z)) * x, s), \alpha_i\,(x, s) / i \in \wedge\}$$

$$= \max\{\inf\{\alpha_i\,((z* (y* z)) * x, s) / i \in \wedge\}, \inf\{\alpha_i\,(x, s) / i \in \wedge\}\}$$
$$= \max\{(\bigwedge_{i\in\wedge}\alpha_i)((z* (y* z)) * x, s), (\bigwedge_{i\in\wedge}\alpha_i)(x, s)\}$$

Therefore $\bigwedge_{i\in\wedge}\alpha_i\,(y * z, s) \leq \max\{(\bigwedge_{i\in\wedge}\alpha_i)((z* (y* z)) * x, s), (\bigwedge_{i\in\wedge}\alpha_i)(x, s)\}$

Hence $\bigwedge_{i\in\wedge}\alpha_i$ strong anti S-fuzzy bi-ideal of Y.

Definition 3.5: Let Y be a BCK-Algebra.Then strong anti S-fuzzy subalgebra of Y if it satisfies: $\alpha(y * z, s) \leq \max\{\alpha(y,s), \alpha(z,s)\}$ for all y, z ∈Y.

Theorem3.6:. Let $\{\alpha_i / i \in I\}$ be any family of strong anti S-fuzzy sub-algebra of Y.Then $\cap_{i\in I}\alpha_i$ and $\cup_{i\in I}\alpha_i$ is a strong anti S- fuzzy sub-algebra of a BCK-Algebra Y.

Proof. Let $\{\alpha_i / i \in I\}$ be any family of strong anti-fuzzy sub-algebra of Yand let y,z ∈ Y

$$\cap_{i\in I}\alpha_i\,(y * z, s) = \inf\{\alpha_i(y* z, s) / i \in I\}$$
$$\leq \inf\{\max\{\alpha_i((y, s), \alpha_i((z, s)\} / i \in I\}$$
$$= \max\{\inf\{\alpha_i((y, s) / i \in I\}, \inf\{\alpha_i(((z, s) / i \in I\}\}$$
$$= \max\{(\cap_{i\in I}\alpha_i)(y, s), (\cap_{i\in I}\alpha_i)(z, s)\}$$

Therefore $\cap_{i\in I}\alpha_i\,(y * z, s) \leq \max\{(\cap_{i\in I}\alpha_i)(y, s), (\cap_{i\in I}\alpha_i)(z, s)\}$.

Hence $\cap_{i\in I}\alpha_i$ is a strong anti S- fuzzy sub-algebra of Y.

Similarly,we can prove that $\cup_{i\in I}\alpha_i$ is a strong anti S- fuzzy sub-algebra of a BCK-Algebra Y.

Theorem 3.7.: Let Y be a BCK-Algebra.Then the fuzzy set α is a strong S- fuzzy sub-algebra of Y iff α^c is a strong anti S-fuzzy sub-algebra of Y.

Proof. Let α be a strong S-fuzzy sub-algebra of Y.

Let y ,z ∈ Y.

$$\alpha^c (y * z, s) = 1 - \alpha (y * z, s)$$
$$= 1 - \min\{ \alpha (y, s), \alpha (z, s)\}$$
$$\leq \max\{1 - \alpha (y, s), 1 - \alpha (z, s)\}$$
$$= \max\{(\alpha^c (y, s), \alpha^c (z, s)\}$$

Therefore $\alpha^c (y * z, s) \leq \max\{\alpha^c (y, s), \alpha^c (z, s)\}$

Conversely, let α^c is a strong anti S-fuzzy sub-algebra of Y.

Let y,z ∈ Y,

$$\alpha (y * z, s) = 1 - \alpha^c (y * z, s)$$
$$\geq 1 - \max\{\alpha^c (y, s), \alpha^c (z, s)\}$$
$$= \min\{1 - \alpha^c (y, s), 1 - \alpha^c (z, s)\}$$
$$= \min\{ \alpha (y, s), \alpha (z, s)\}$$

Therefore $\alpha (y * z, s) \geq \min\{ \alpha (y, s), \alpha (z, s)\}$

Hence α is a strong S- fuzzy sub-algebra of Y.

Theorem 3.8: An anti-homomorphic pre-image of an anti strong S-fuzzy bi-ideal is an anti strong S- fuzzy bi-ideal of a BCK-Algebra Y.

Proof. Let f : Y → Z be an anti homomorphism of a bck-Algebra Y.

Let β be a strong S fuzzy bi-ideal of Y .

Let x, y, z ∈ Y.

$$f^{-1}(\beta)(y * z, s) = \beta (f(y * z, s))$$
$$= \beta (f(y) * f(z), s)$$
$$\leq \max\{ \beta (f(((z* (y* z)) * x, s)), \beta (f(x, s)) \}$$
$$= \max\{f^{-1} (\beta) ((z* (y* z)) * x, s), f^{-1} (\beta)(x, s)\}$$

Therefore $f^{-1} (\beta) (y * z, s) \leq \max\{f^{-1} (\beta) ((z* (y* z)) * x, s), f^{-1} (\beta)(x, s)\}$

Hence $f^{-1} (\beta)$ is an anti strong S -fuzzy bi-ideal of Y.

Theorem 3.9: An anti homomorphic image of a strong anti S- fuzzy bi-ideal with sup property is a strong anti S- fuzzy bi-ideal of a BCK-Algebra Y.

Proof. Let f : Y → Z be an anti homomorphism of a bck-Algebra Y.

Let α be a strong anti S- fuzzy bi-ideal with sup-property.

Let β be the image of α .

Let f(x), f(y), f(z) ∈ f(Y).

Let $(x_0, s) \in f^{-1}(f(x,s)), (y_0, s) \in f^{-1}(f(y,s)), (z_0, s) \in f^{-1}(f(z,s))$ such that

$$\alpha(x_0, s) = \sup_{t \in f^{-1}(f(x,s))} \alpha (t, s),$$
$$\alpha (y_0, s) = \sup_{t \in f^{-1}(f(y,s))} \alpha (t, s),$$
$$\alpha (z_0, s) = \sup_{t \in f^{-1}(f(z,s))} \alpha (t, s).$$
$$\beta (f(y) * f(z), s) = \beta (f(y * z, s))$$
$$= \sup_{t \in f^{-1}(f((y * z, s))} \alpha (t, s),$$
$$\leq \alpha (y_0 , z_0 , s)$$
$$\leq \max\{\alpha ((z_0* (y_0*z_0)) *x_0, s) \alpha(x_0 , s),\}$$
$$= \max\{\sup_{t \in f^{-1}(f(z * (y * z)) * x, s)} \alpha (t, s), \sup_{t \in f^{-1}(f((x, s))} \alpha (t, s) \},$$
$$= \max\{\beta (f((z* (y * z)) * x, s), \beta (f(x, s))\}$$

Therefore, $\beta (f(y) * f(z), s) \leq \max\{\beta (f((z* (y* z)) * x, s), \beta (f(x, s))\}$

Hence β is a strong anti S- fuzzy bi-ideal of Z .

Theorem 3.10: Let f : Y → Z be an onto homomorphism of a BCK-Algebra Y.If β is a strong S- fuzzy bi-ideal in Z , then $f^{-1}(\beta)$ is a strong S- fuzzy bi-ideal in a BCK-Algebra Y.

Proof. Let β be a strong S fuzzy bi-ideal of Z .

For all x,y,z ∈ Y, $f^{-1}(\beta)(y* z, s) = \beta (f(y* z, s))$

$$= \beta (f(y) * f(z), s)$$
$$\geq \min\{ \beta ((f(z) * f(y*z)) * f(x), s), \beta (f (x), s)\}$$

$=\min\{ f^{-1}(\beta)((z* (y* z)) * x, s), f^{-1}(\beta)(x, s)\}$

Therefore , $f^{-1}(\beta)(y* z, s) \geq \min\{ f^{-1}(\beta)((z* (y* z)) * x, s), f^{-1}(\beta)(x, s)\}$

Hence $f^{-1}(\beta)$ is a strong S-fuzzy bi-ideal in a BCK-Algebra Y.

Theorem 3.11: Let Y be a BCK-Algebra and α be a fuzzy set in Y. Then α is a strong anti S-fuzzy bi-ideal in Y iff α^c is a strong S-fuzzy bi-ideal in Y.

Proof. Let Y be a BCK-Algebra and α be a strong anti S-fuzzy bi-ideal in Y.

For x, y, z ∈ Y,

$$\alpha^c(y * z , s) = 1 - \alpha(y * z , s)$$
$$\geq 1 - \max\{ \alpha((z* (y* z)) * x, s), \alpha(x, s)\}$$
$$= \min\{1 - \alpha((z* (y* z)) * x, s), 1- \alpha(x ,s)\}$$
$$= \min\{ \alpha^c((z* (y* z)) * x, s), \alpha^c(x, s)\}$$

Therefore $\alpha^c(y * z , s) \geq \min\{\alpha^c((z* (y* z)) * x, s), \alpha^c(x, s)\}$

Hence α^c is a strong S-fuzzy bi-ideal in Y.

Conversely suppose that α^c is a strong fuzzy bi-ideal in Y.

For any x, y, z ∈ Y,

$$\alpha(y* z, s) = 1 - \alpha^c(y * z, s)$$
$$\leq 1 - \min\{\alpha^c((z* (y* z)) * x, s), \alpha^c(x, s)\}$$
$$= \max\{1 - \alpha^c((z* (y* z)) * x, s), 1-\alpha^c(x, s)\}$$
$$= \max\{ \alpha((z* (y* z)) * x, s), \alpha(x , s)\}$$

Therefore $\alpha(y * z, s) \leq \max\{ \alpha((z* (y* z)) * x, s), \alpha(x, s)\}$.

Hence α is a strong anti fuzzy bi-ideal in Y.

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